5

SERVICE LOAD DEFLECTION

5.1 GENERAL INTRODUCTION

Prestressed concrete beams are generally more slender (smaller E_cI) than their reinforced concrete counterparts because of two main reasons – use of higher strength materials, both concrete and steel, and the effect of balancing part of the external loads due to prestress. As deflection is inversely proportional to the flexural rigidity, E_cI, slender prestressed concrete beams are susceptible to deflect more for a given load and, hence, demands special design attention to avoid any associated serviceability problems.

While dealing with in-service deflection of a beam, it is helpful to identify the various types of loads that the beam is intended to carry and classify them according to the duration of their action, generally as, permanent (or sustained) and temporary (or transitory). This classification is necessary because deflection produced by permanent loads keeps on increasing with time due to time-dependent response of the constituent materials. In the case of a prestressed concrete member, the loads and their classification are shown in Table 5.1

Table 5.1 Loads and their classification

Load	Classification	Comments
Prestressing force, F_i	Permanent (or sustained)	F_i reduces with time to a final value, F_e
Dead load: Self weight Other loads	Permanent	
Imposed load	25% Permanent 75% Temporary (or transitory)	Part of the imposed load is treated as permanenet

It may be noted in table 5.1 that for normal residential or office occupancy, BS 8110 specifies that 25 % of the imposed load should be considered as permanent or sustained and the remainder may be considered as transitory in nature for the calculation of long-term deflection. However, for storage areas, when an upper limit to the final long-term deflection is required, 75% of the imposed load should be considered as permanent

With regard to service load deflection, the major difference between reinforced and prestressed concrete beams is that the application of prestress generally produces an upward deflection or *camber*. This camber tends to decrease with time as a result of the continuous loss of prestress due to shrinkage of concrete and relaxation of steel. However, the effect of the creep of concrete is of twofold. Although, it produces a loss of prestress tending to reduce the camber, creep strains in the concrete usually increase the negative curvature of the beam associated with prestress and, hence, increase the camber.

Both self-weight and imposed loads usually produce downward deflection that superimposes on the upward deflection due to prestress. The deflection produced by the sustained part of these external loads also increases with time due to concrete creep. Thus, camber due to prestress is beneficial to the total long-term deflection of a beam because the applied load must negate the camber before producing any downward deflection. However, like excessive deflection, large initial camber may lead to several serviceability problems, such as, improper drainage of roof decks, cracking of partitions or other non-structural elements, ill-fitting doors and windows, and an uneven floor surface. Hence, in addition to limiting the deflection, explicit attention needs to be paid to control initial camber for a prestressed concrete beam, particularly those of Class 1 and Class 2 categories.

The calculation of deflection of a prestressed concrete beam requires a good understanding of deflection calculation for homogeneous elastic beams. Although elastic deflection is usually covered in the structural mechanics course, a brief review is presented herein to facilitate ready recollection of the basic principles.

5.2 DEFLECTION OF ELASTIC BEAMS

For the beam and co-ordinate system shown in Fig. 5.1, and assuming linear elastic behaviour, the elementary mechanics gives the following second-order differential equation for deflection, Δ, at any distance, x from the origin:

$$\frac{d^2\Delta}{dx^2} = \frac{M(x)}{EI} = \frac{1}{r_x} \qquad (5.1)$$

in which EI is the flexural rigidity of the beam section and $M(x)$ is the moment at a distance, x from the origin, sagging moment being considered positive, and r_x is the radius of curvature of the elastic curve at x.

Knowing the loading and the geometry of a beam, $M(x)$ can be expressed in terms of the magnitude and location of the applied loads. Deflection can then be calculated simply by double integration of Eq. (5.1). The unknown constants of integration are determined from the relevant boundary conditions.

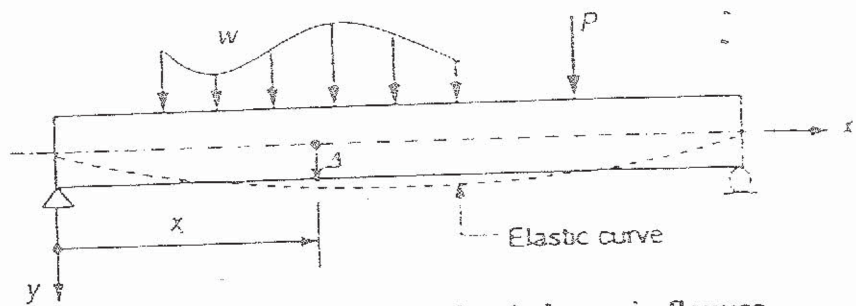


Fig. 5.1 Deflection of simple beam in flexure.

Alternatively, simple methods like *moment-area method or conjugate beam method* may be conveniently used. For ready recollection, the deflection theorem of the area-moment method can be stated as follows:

The deflection of any point i of a flexural member measured with respect to the tangent to the elastic curve (deflected shape) of the member at any other point j is equal to the first static moment taken about i of the area under M/EI diagram along the member taken between points i and j.

It is to be noted that the deflection is measured with respect to the tangent to the elastic curve. Therefore, appropriate choice of the reference tangent will substantially reduce the calculation. This is illustrated in Example 5.1.

EXAMPLE 5.1: DEFLECTION OF ELASTIC BEAMS

Calculate the maximum deflection of a simply supported beam of uniform cross section due to uniformly distributed load, w, as shown in Fig. E5.1 by (a) integration of Eq. (5.1) and (b) moment-area method. Ignore self-weight of the beam.

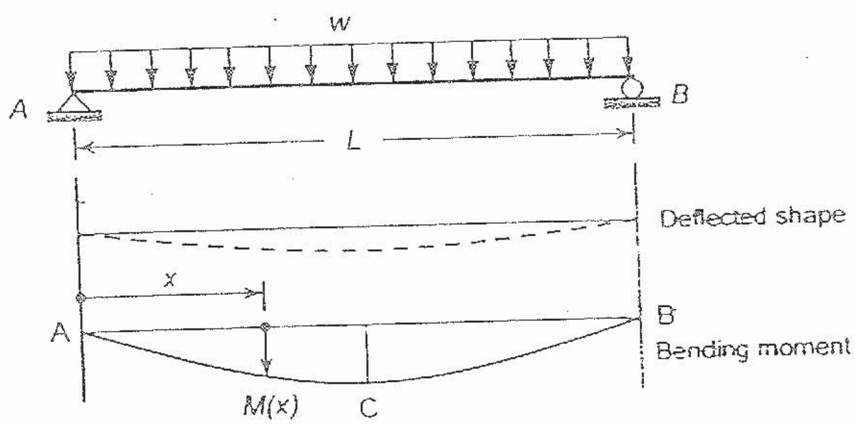


Figure E5.1.1 Beam in Example E5.1.

SOLUTION

(a) Method of integration

The expression for moment at any distance, x from the support A (Fig. E5.1.1) may be obtained as

$$M(x) = \frac{wLx}{2} - \frac{wx^2}{2} \qquad \text{(E5.1a)}$$

Substituting in Eq. (5.1), we write

$$EI\frac{d^2\Delta}{dx^2} = \frac{wL}{2}x - \frac{wx^2}{2} \qquad \text{(E5.1b)}$$

Integrating twice with respect to x, we obtain

$$EI\frac{d^2\Delta}{dx^2} = -\frac{wx^4}{24} + \frac{wLx^3}{12} + C_1x + C_2 \qquad \text{(E5.1c)}$$

in which C_1 and C_2 are constants of integration.

Introducing the following boundary conditions

$$\Delta = 0 \quad \text{at} \quad x = 0, \text{ and}$$
$$\Delta = 0 \quad \text{at} \quad x = L$$

we obtain

$$C_2 = 0 \quad \text{and} \quad C_1 = -\frac{wL^3}{24}$$

Inserting these constants into Eq. (E5.1c), we obtain the expression for deflection as

$$\Delta = \frac{w}{24EI}\left(-x^4 + 2Lx^3 - L^3x\right) \qquad \text{(E5.1d)}$$

The maximum deflection will occur at midspan. Letting $x = L/2$ in Eq. (E5.1d), we have

$$(\Delta)_{max} = \frac{5wL^4}{384EI} \downarrow$$

(b) Moment-area method

The *M/EI* diagram for the given loading is shown in Fig. E5.1.2(a), while Fig. E5.1.2(b) shows the deflected shape (elastic curve) of the beam. It is obvious that the tangent to the elastic curve is horizontal at midspan, and deflection of point A with respect to this tangent represents the maximum deflection of the beam. Therefore, the reference point *i* will be taken at support A, and *j* at midpan C. According to the second moment-area theorem, the required maximum deflection of the beam is equal to the moment of the hatched portion of the *M/EI* diagram (Fig. E5.1.2a) about the point A.

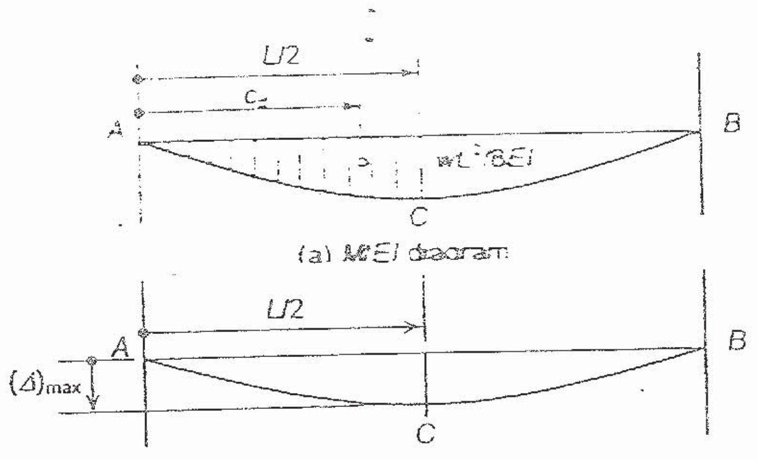


Figure E5.1.2 Calculation of deflection by moment-area method.

The area of the hatched segment of the parabolic M/EI is given by

$$A = \frac{2}{3} \times (\text{base}) \times (\text{altitude}) = \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8EI} = \frac{wL^3}{24EI}$$

The distance of the centroid of this area is given by

$$c_d = \frac{5}{8} \times (\text{base}) = \frac{5}{8} \times \frac{L}{2} = \frac{5}{16} L$$

Therefore

$$(\Delta)_{max} = \frac{wL^3}{24EI} \times \frac{5L}{16} = \frac{5wL^4}{384EI} \downarrow$$

5.3 DEFLECTION OF PRESTRESSED CONCRETE BEAMS

According to the definition, fully prestressed concrete members (Class 1 and Class 2) remain crack-free under service load and, hence, can be assumed to be linearly elastic. Since the deflection under service load is of concern, the methods briefly described above can be used for prestressed concrete beams as well. However, it is necessary to distinguish between short-term or immediate or instantaneous deflection and long-term or time-dependant deflection. Instantaneous deflection occurs immediately upon the application of a load. If this load is held for a long time, the beam will keep on deflecting and the magnitude of deflection will become asymptotic to a final value at time infinity. The total deflection experiences by a beam can be separated into two parts: an instantaneous elastic part and an additional time-dependent part, as shown in Fig. 5.2. The methods available for calculating deflection in elastic beams can be used to assess instantaneous deflections only. Calculation of additional deflection needs integration of long-term material response.

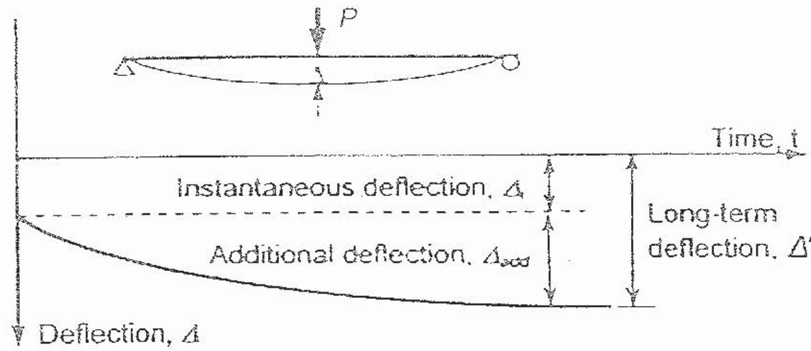


Figure 5.2 Deflections of a beam due to sustained loading.

5.4 INSTANTANEOUS DEFLECTIONS

In applying the methods of elastic beams to those of prestressed concrete, two important properties must be defined, namely, the modulus of elasticity of the concrete and moment of inertia of the section.

The modulus of elasticity of the concrete material can be estimated from the recommendations given in the Code. However the strength of the concrete varies with age, and so does its modulus. In computing initial camber or deflection, it is common to use the initial modulus E_{ci} while E_c is considered for service load deflections.

With regard to the moment of inertia, it is customary to use the gross moment of inertia I_g for pretensioned members and the net moment of inertia I_n for post-tensioned members with unbonded tendons. In the case of bonded tendons, the moment of inertia of the transformed section can be used. However, often it does not lead to a significant gain in accuracy when compared to that obtained by using I_g.

5.4.1 Due to External Loads

Provided appropriate values of E_c and I are used, any of the methods discussed in Article 5.3 may be used to calculate the instantaneous deflection of a prestressed concrete beam due to any external loading. For usual loading and support conditions, standard formulae are available in any handbook on structural engineering. Typical formulae for several types of loading are given in Fig. 5.3.

5.4.2 Due to Prestress

As discussed in Chapter 2, the effect of prestressing can be replaced by a set of external loads acting on the beam. Once the equivalent loads are known, the moment produced by these loads can be expressed as a function of x, and Eq. (5.1) be used to calculate the resulting deflection at any point. However, for a simply supported beam, the prestress moment diagram is directly proportional to the eccentricity diagram, which is generally a function of x, that is $M(x) = F_i e_o(x)$. Thus, there should not be any difficulty in applying either Eq. (5.1) or the moment-area method to calculate the instantaneous deflection of the beam due to prestress alone. Example 5.2 illustrates the methods, and formulae for typical cable profiles are given in Fig. 5.3.

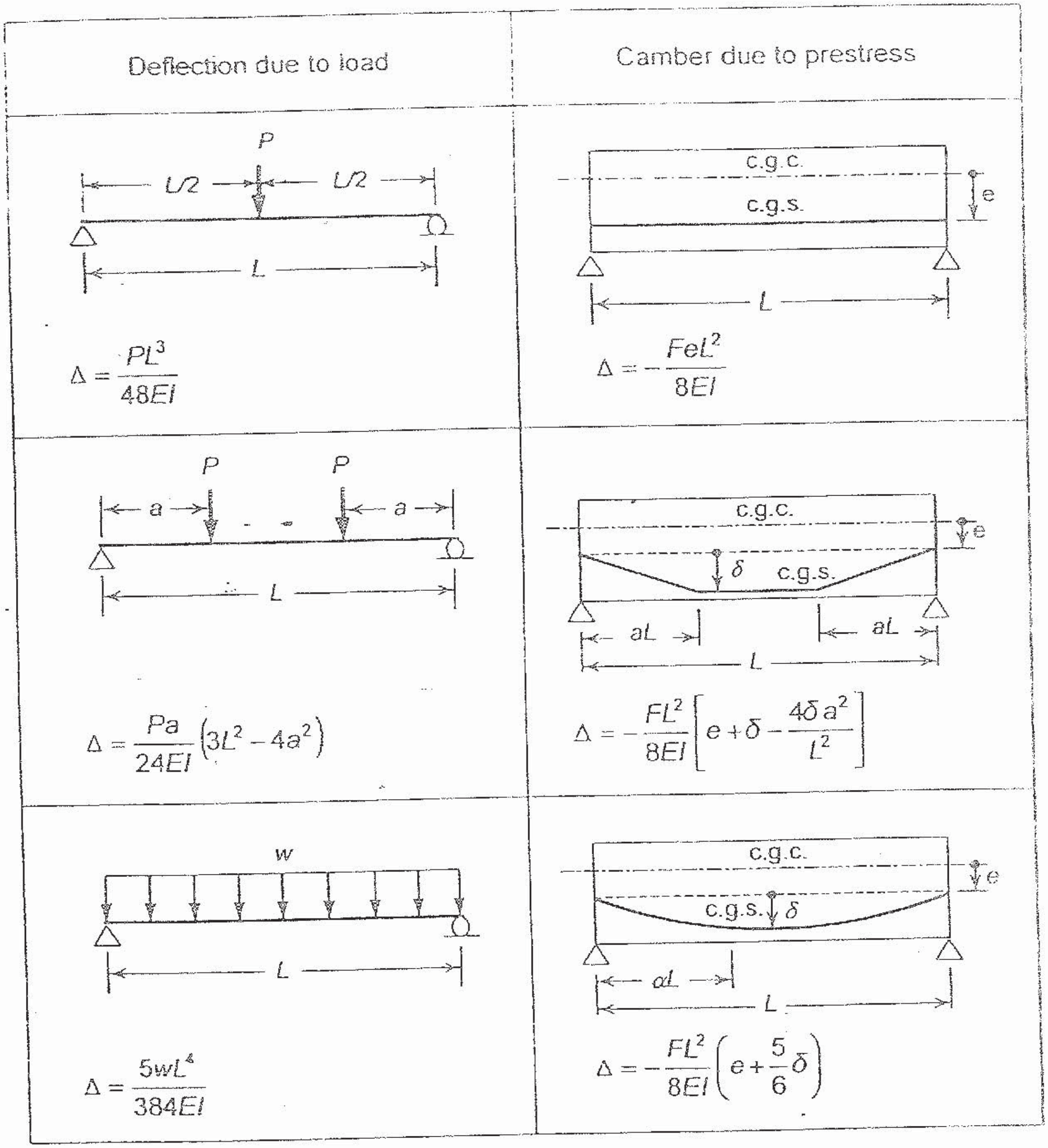


Figure 5.3 Expressions for short-term midspan deflections due to typical loads and tendon profiles.

EXAMPLE 5.2 : DEFLECTION DUE TO PRESTRESS ALONE

Determine the deflection (camber) for a simply supported beam of uniform cross section due to prestressing a draped tendon, as shown in Fig. E 5.2.(a). No external load, including its own weight is to be considered.

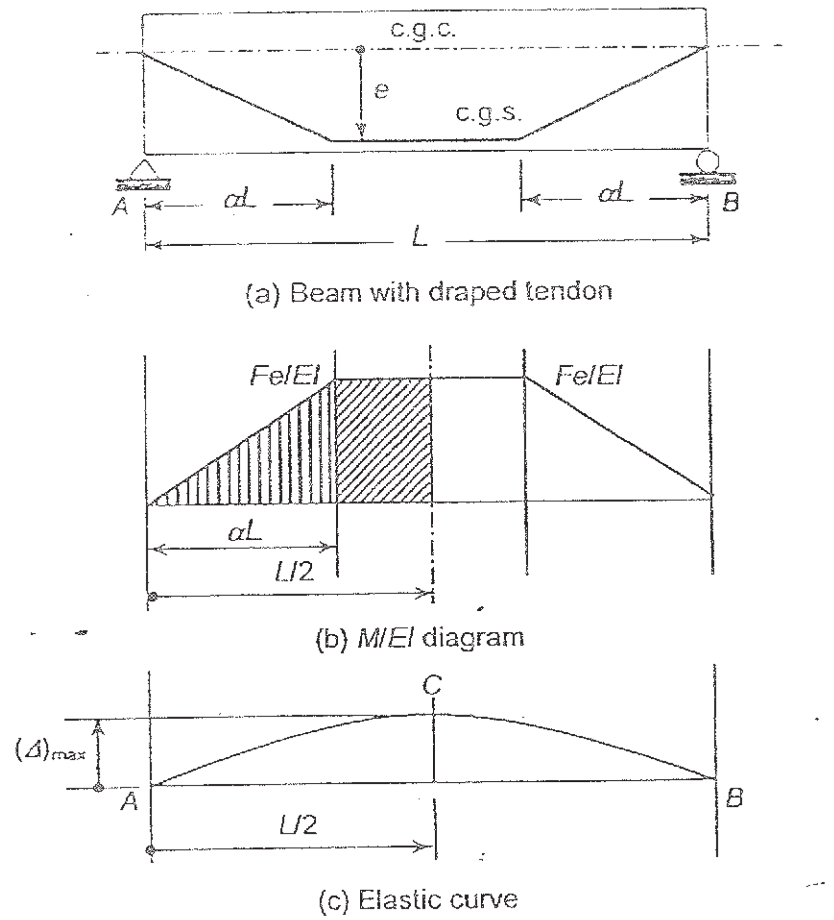


Figure E5.2 Calculation of deflection due to prestress.

SOLUTION

The *M/EI* diagram for prestressing the draped tendon in Fig. E5.2(a) is shown in Fig. E5.2(b). Since the moment is negative, the beam will deflect upward as shown in Fig. E5.2(c). Obviously, the tangent to the elastic curve is horizontal at midspan, and deflection of point A with respect to this tangent represents the maximum deflection of the beam. Taking the reference point *i* at support A, and *j* at midspan C, the required maximum deflection of the beam will be equal to the moment of the hatched portion of the *M/EI* diagram (Fig. E5.2b) about the point A. For convenience, this portion is divided into a component triangle and a rectangle, as shown. Thus,

$$(\Delta)_{max} = \frac{1}{2} \times \alpha L \times \frac{Fe}{EI} \times \frac{2}{3} \times \alpha L + \left(\frac{L}{2} - \alpha L\right) \times \frac{Fe}{EI} \times \left[\alpha L + \left(\frac{L}{2} - \alpha L\right) \times \frac{1}{2}\right]$$

$$= \frac{FL^2}{8EI}\left(e - \frac{4\alpha^2}{3}e\right) \uparrow$$

EXAMPLE 5.3: CALCULATION OF INITIAL CAMBER

A simply supported pre-tensioned concrete beam spans 10 m and has a rectangular cross-section, 400 mm wide and 600 mm overall depth. The tendons have a parabolic profile with eccentricities of zero at both ends and 200 mm at midspan. If the tendons are stressed to furnish an initial prestressing force, F_i of 1315 kN at midsapn, calculate the camber immediately after the transfer of prestress. Assume E_{ci} = 28 GPa and neglect the effect of friction.

SOLUTION

Moment of inertia of the section

$$I = \frac{400 \times 600^3}{12} = 7.2 \times 10^9 \text{ mm}^4$$

Self-weight, $w_G = 0.4 \times 0.6 \times 24 = 5.76$ kN/m

At transfer of prestress, the self-weight of the beam will be activated. Prestress will produce upward deflection (camber), whereas self-weight will give downward deflection. The net deflection will be the algebraic summation of the two. Using the relevant expressions from Fig. 5.3, we get

$$\Delta = -\frac{FL^2}{8EI}\left(\frac{5}{6}\delta\right) + \frac{5w_G L^4}{384EI}$$

where δ is the cable sag at midspan.

In the present problem,

δ = 200 mm;	$E = E_{ci}$ = 28 GPa:	$I = 7.2 \times 10^9$ mm^4;
F = 1315 kN;	L = 10 m;	w_G = 5.76 kN/m.

Inserting these values into the above formula for elastic deflection, we obtain

$$\Delta = -\frac{1315 \times 10^3 \times 10^2 \times 10^6}{8 \times 28{,}000 \times 7.2 \times 10^9} \times \frac{5}{6} \times 200 + \frac{5 \times 5.76 \times 10^4 \times 10^{12}}{384 \times 28{,}000 \times 7.2 \times 10^9}$$

$$= -13.6 + 3.7 = -9.9 \text{ mm} \uparrow$$

5.5 LONG-TERM DEFLECTION

The instantaneous deflections produced by prestress, dead load, including the self-weight and sustained part of the imposed load are continuously modified due to the time-dependent response of the constituent materials, that is, creep, shrinkage and steel relaxation. Before discussing their effects on long-term deflections of a beam, it may be helpful to briefly treat the case of concrete creep for an element.

Fig. 5.4 shows the increase in creep strain, ε_c, with time due to a constant sustained stress, σ_o applied to a concrete element at age T_o. Creep strain, $\varepsilon_c(t)$, at any subsequent time T_1 depends primarily on (a) age of concrete at first loading, (b) duration of loading, (c) ambient conditions, and (d) shape and size factor of the member. The total strain in the element at time infinity is shown in Fig. 5.4(c). It consists of the elastic (instantaneous) strain, ε_i, and creep strain, ε_c' at time infinity, so that the total strain is given by

$$\varepsilon' = \varepsilon_i + \varepsilon_c' \tag{5.2}$$

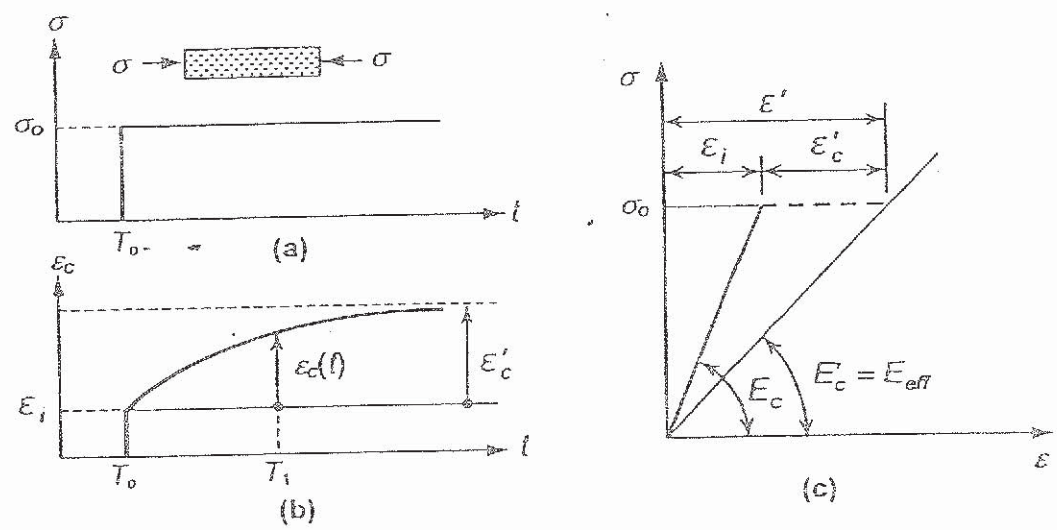


Fig. 5.4 Elastic and creep strains in an element (a) stress history, (b) strain history, and (c) long-term total strain.

Creep function and creep coefficient

For practical purposes, creep strain can be assumed to be proportional to the level of stress throughout the working load range. That is,

$$\varepsilon_c(t) \propto \sigma_o \tag{5.3}$$

The instantaneous strain is also proportional to σ_o, that is,

$$\varepsilon_i \propto \sigma_o \tag{5.4}$$

Combining Eqs. (5.3) and (5.4), we obtain

$$\varphi(t) = \frac{\varepsilon_c(t)}{\varepsilon_i} \tag{5.5}$$

The ratio of creep strain to instantaneous elastic stain as defined by Eq. (5.5) is known as the *creep function*, $\varphi(t)$. It is independent of the applied stress, σ_o, but is dependent on many factors, the primary ones being the environmental conditions, age of concrete at first loading and shape and size of the member

At time infinity, creep function reduces to

$$\varphi = \frac{\varepsilon'_c}{\varepsilon_i} \tag{5.6}$$

Known as the *creep coefficient*, φ is a very useful measure of the potential creep of the concrete, which can be easily determined from experiments. For design purpose, the long-term creep coefficient may be obtained from Fig. 4.9.

Long-term strain and deflection

Substituting ε'_c from Eq. (5.6) into Eq. (5.2), we can obtain the total long-term stain in the element as a function of elastic stain and the creep coefficient as

$$\varepsilon' = \varepsilon_i(1+\varphi) \tag{5.7}$$

Thus, the total long-term strain in the element may simply be obtained by magnifying the elastic instantaneous stain, ε_i by a factor $(1 + \varphi)$.

The long-term defection of a beam is analogous to the strain in the element as discussed above. Therefore,

$$\Delta' = \Delta_i(1+\varphi) \tag{5.8}$$

in which Δ' is the long-term deflection, Δ_i is the instantaneous deflection, and $(1+ \varphi)$ is the magnification factor.

5.5.1 Due to Permanent External Loads

The dead loads, which include self-weight of the beam and weights of finishes, fixtures and partitions, are permanent loads. Also, a part of the imposed load, such as fixed or movable furniture and equipment act permanently on the structure. It is therefore necessary to make an assessment of which fraction of the applied loads may be considered as sustained and at what age of concrete each of these loads are applied to enable an accurate estimate of the long-term creep effect. Sometimes, it may be necessary to estimate the creep deflection at intermediate stages of loading. In such a case, an accurate description of the creep function, $\varphi(t)$ is also necessary. One such typical creep function for moist cured members is given as follows:

$$\varphi(t) = \left[\frac{t^{0.6}}{10 + t^{0.6}}\right]\varphi K_{CH} K_{CA} K_{CS} \tag{5.9}$$

in which

K_{CH} = correction factor for humidity = $1.27 - 0.006H$

K_{CA} = factor for age at loading = $1.25t_A^{-0.118}$

K_{CS} = shape and size factor = $1.14 - 0.9(V/S)$

where t_A is the age at loading, H is the relative humidity in percent, V/S is the volume-to-surface ratio of the member.

As discussed in the preceding section, the total deflection at any time t for a sustained load may obtained as

$$\Delta(t) = \Delta_i [1 + \varphi(t)] \tag{5.10}$$

It may be noted in Eq. (5.8) that the calculation of total deflection due to a sustained load consists of a fixed component, Δ_i and an additional component, Δ_{add} as given by

$$\Delta_{add} = \Delta_i \varphi(t) \tag{5.11}$$

It involves calculation of the instantaneous deflection, Δ_i and estimation of $\varphi(t)$, the creep coefficient at the desired time t. For a combination of sustained loads applied at different ages, long-term deflection for each load may be added algebraically to obtain the total deflection.

5.5.2 Due to Prestress

It has been pointed out that the application of the initial prestressing force, in general, produces an immediate upward deflection or camber and that this initial camber is continuously modified due to the combined effects of shrinkage and creep of concrete, and relaxation of steel. While shrinkage and relaxation tend to decrease the initial camber due to the resulting loss of prestress, the effect of concrete creep of concrete is twofold. Although, it produces a loss of prestress tending to reduce the camber, creep strains in the concrete usually increase the negative curvature of the beam associated with prestress and, hence, increase the camber. In order to have a better understanding, it is convenient to consider the curvature after losses of prestress, ψ_{pe} as the sum of three components:

(a) the instantaneous curvature, ψ_{pi} occurring immediately upon application of F_i,

(b) the change in curvature $d\psi_1$ corresponding to loss of prestress due to shrinkage, creep, and relaxation, and

(c) the change in curvature $d\psi_2$ resulting from the direct effect of concrete creep under sustained compression.

Thus,

$$\psi_{pe} = \psi_{pi} + d\psi_1 + d\psi_2 \tag{5.12}$$

In Eq. (5.1), $d^2\Delta / dx^2$ represents the curvature of the section, and for statically determinate beams, prestress moment diagram is directly proportional to the eccentricity diagram, that is, $M = -Fe_o$. Therefore, Eq. (5.1) may be rewritten as

$$\psi = \frac{M}{E_c I} = \frac{-Fe_o}{E_c I} \tag{5.13}$$

Assuming that creep occurs under a constant prestressing force, equal to the average of the initial and final values, and that the modulus of elasticity of the concrete is represented by the average of E_o and E_c, that is $(E_c)_{av}$. Eq. (5.12) may be restated as

$$\psi_{pe} = -\frac{F_i (e_o)_x}{(E_c)_{av} I} + (F_i - F_e)\frac{(e_o)_x}{(E_c)_{av} I} - \left(\frac{F_i + F_e}{2}\right)\frac{(e_o)_x}{(E_c)_{av} I}\varphi \qquad (5.14)$$

The subscript x used for e_o indicates that the eccentricity varies along the span. In Eq. (5.14), the first term is the initial negative curvature, the second term is the reduction in that initial curvature because of the losses of prestress, and the third term is the increase in negative curvature because of the creep of concrete. Using these curvatures as the elastic load on the beam, the corresponding deflections may be obtained by the moment-area method, formulae for typical tendon profiles being given in Fig. 5.3. The final deflection of the member under the action of F_e is therefore

$$(\Delta')_{pe} = -\Delta_{pi} + (\Delta_{pi} - \Delta_{pe}) - \left[\frac{\Delta_{pi} + \Delta_{pe}}{2}\right]\varphi \qquad (5.15)$$

which reduces, after simplification, to

$$(\Delta')_{pe} = -\Delta_{pe} - \left[\frac{\Delta_{pi} + \Delta_{pe}}{2}\right]\varphi \qquad (5.16)$$

where the first term may be obtained by direct proportion as:

$$\Delta_{pe} = \Delta_{pi}\frac{F_e}{F_i} \qquad (5.17)$$

Thus, the long-term deflection (camber) due to prestress alone may be obtained from Eq. (5.16) provided that the instantaneous deflection due to initial prestess and the value of the creep coefficient are known.

The method described above is approximate in nature because it involves a number of simplifying assumptions, and ignores the interaction of the effects of creep, shrinkage and relaxation. A more accurate means of assessing the time-dependent deflection is to account for the incremental changes in the material properties and the prestressing force in a series of discrete time steps. However, such a refinement increases the analytical complexities, and is rarely practiced in all but delicate cases.

5.6 TOTAL DEFLECTIONS

In order to calculate the total deflection (or camber) at a particular stage of loading, it is first of all necessary to identify the various loads that have been applied to the beam, the nature of each load (whether it is sustained or temporary), and the age of concrete at which each load is applied. Once this is done, each load may be considered separately and the resulting deflection, including the additional deflection due to time-dependent material response, if any, can be easily calculated using the procedure described above. Fig. 5.5 shows the components of deflections for the entire beam. The maximum values of the total or incremental deflections can then be obtained by linear superposition of relevant deflections.

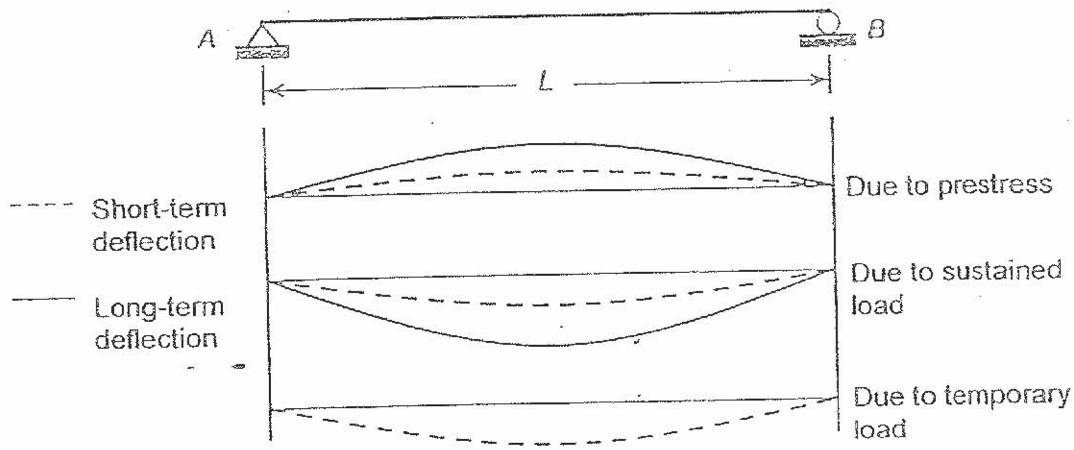


Figure 5.5 Components of deflection.

5.7 DEFLECTION LIMITS

Structural members, despite having sufficient strength, may develop excessive cambers or deflections over time that can jeopardise their in-service performance. Hence, explicit design attention needs to be given to limit the deflection or camber to some permissible values.

The British code provides deflection limitations for building members basically to ensure an acceptable appearance of the structure and to protect the non-structural elements from damage by cracking, crushing or spalling. In the latter case, the limits imposed depend on whether the non-structural elements are brittle or ductile in nature. For brittle material, the provision is quite stringent. The indicative deflections suggested in the code are summarised in Table 5.1.

Table 5.1 Maximum permissible deflections according to BS 8810-97.

Considerations	Type of member	Deflection to be considered	Deflection limitation
Appearance	Visible member	Total long-term deflection, $(\Delta)_{total}$	L/250
Damage to non-structural elements	Brittle partitions or finishes	Deflection after installation of non-structural member, $(\Delta)_{inc}$	L/500 or 20 mm
	Non-brittle partitions or finishes		L/350
NOTE: - *L* is the span or, in the case of a cantilever, its length - These values also apply to upward deflection.			

EXAMPLE 5.4

The cross-section of a post-tensioned concrete beam, simply supported over a span of 28 m, is shown in Fig. E5.4. The beam is to carry a uniformly distributed characteristic dead load of 4 kN/m, in addition to its own weight, and a characteristic imposed load of 10 kN/m, 25% of which should be treated as permanent. The required initial prestressing force of 2610 kN was furnished by stressing 14 numbers of 15.7 mm diameter, 7-wire stands (A_{ps} = 150 mm^2), which was transferred to the concrete at the age of 7 days. The prestressing steel is contained equally in two cables having parabolic profiles with eccentricities of zero at each end and 762 mm at mid-span. The following information is given:

f_{ci} = 30 MPa;	E_{ci} = 26 GPa;	
f_{cu} = 50 MPa;	E_c = 31GPa	
f_{pu} = 1770 MPa,	E_{ps} = 200 GPa	
A_c = 508,000 mm^2;	$I = 134 \times 10^9$ mm^4;	
y_t = 588 mm;	y_b = 912 mm;	η = 0.8

Assuming that 60 percent of the superimposed dead load was due to brittle non-structural components installed at the age of 28 days, and that the structure is exposed to an average relative humidity of 75%, check the serviceability limit state of deflection for the beam.

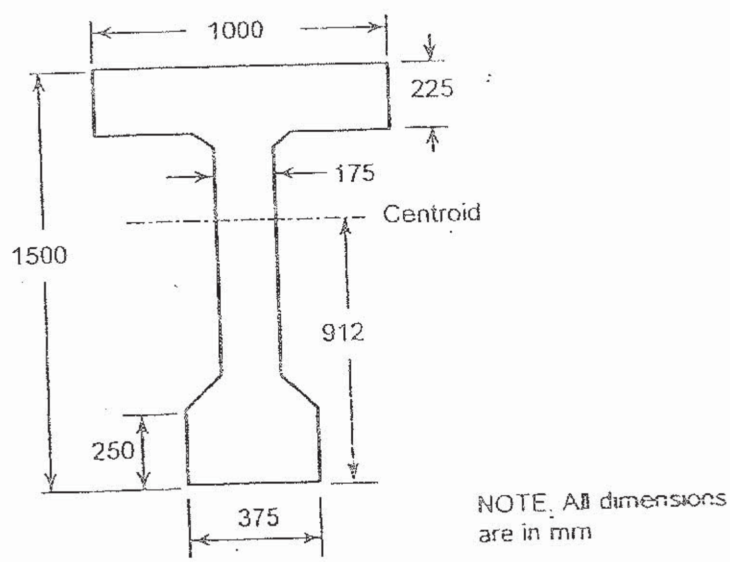


Figure E5.4 Section details.

SOLUTION

(a) Preliminary calculations

Self-weight of the beam = 0.508 × 24 = 12.19 kN/m

Assuming that the beam is precast, and all surfaces are exposed to the environment, the effective section thickness, ignoring the fillets, is

$$t_e = \frac{2A_c}{\text{Exposed perimeter}}$$

$= (2 \times 508{,}000)/(2 \times 1000 + 2 \times 1500 + 2 \times 200)$

≈ 190 mm

The creep co-efficient corresponding to a relative humidity of 75% and an effective section thickness of 190 mm may be obtained by linear interpolation of the values obtained from Fig. 4.9 as

$\varphi_7 = 2.54$ (for age at loading of 7 days)

$\varphi_{28} = 2.10$ (for age at loading of 28 days)

(b) Instantaneous deflections

Assume that the prestress was transferred at 7 days and that the instantaneous deflections (camber) due to initial prestress, self-weight, and other permanent and temporary loads are calculated separately as follows:

Due to initial prestress, F_i

$$\Delta_{pi} = -\frac{F_i L^2}{8E_{ci}I}\left(\frac{5}{6}\delta\right) = -\frac{2610 \times 10^3 \times 28{,}000^2}{8 \times 26{,}000 \times 134 \times 10^9} \times \frac{5}{6} \times 762 = -46.6 \text{ mm}\uparrow$$

Due to self-weight

$$(\Delta_i)_{SW} = \frac{5w_G L^4}{384E_{ci}I} = \frac{5 \times 12.19 \times 28{,}000^4}{384 \times 26{,}000 \times 134 \times 10^9} = 28.0 \text{ mm}\downarrow$$

All other loads are assumed to be applied at 28 days. The permanent and temporary components of these loads are as follows:

Permanent load = 4 + 0.25 × 10 = 6.5 kN/m

Temporary load = 10 – 2.5 = 7.5 kN/m

Thus,

$$(\Delta_i)_{PL} = \frac{5wL^4}{384E_c I} = \frac{5 \times 6.5 \times 28{,}000^4}{384 \times 31000 \times 134 \times 10^9} = 12.5 \text{ mm}\downarrow$$

and

$$(\Delta_i)_{TL} = \frac{12.5 \times 7.5}{6.5} = 14.4 \text{ mm}\downarrow$$

(c) Long-term deflections

Due to prestress

$$\Delta_{pe} = \Delta_{pi}\frac{F_e}{F_i} = -46.6 \times \frac{0.8 \times 2610}{2610} = -37.3 \text{ mm}\uparrow$$

$$(\Delta')_{pe} = -\Delta_{pe} - \left[\frac{\Delta_{pi} + \Delta_{pe}}{2}\right]\varphi = -37.3 - \left[\frac{46.6 + 37.3}{2}\right] \times 2.54 = -144\ \text{m}\uparrow$$

Due to self-weight

$$(\Delta')_{SW} = (\Delta_i)_{SW}(1+\phi) = 28 \times (1+2.54) = 99.1\,\text{mm}\downarrow$$

Due to other permanent loads

$$(\Delta')_{PL} = (\Delta_i)_{PL}(1+\varphi) = 12.5 \times (1+2.1) = 38.8\,\text{mm}\downarrow$$

(d) Check for serviceability limit state of deflection

Initial camber

$$\Delta_i = \Delta_{pi} + (\Delta_i)_{SW}$$
$$= -46.6 + 28 = -18.6\ \text{mm}\uparrow$$
$$< \frac{L}{250} = \frac{28{,}000}{250} = 112\,\text{mm} \quad \text{O.K.}$$

Total deflection

$$(\Delta)_{total} = (\Delta')_{SW} + (\Delta')_{PL} + (\Delta_i)_{TL} + (\Delta')_{pe}$$
$$= 99.1 + 38.8 + 14.4 - 144 = 8.3\ \text{mm}\downarrow$$
$$< \frac{L}{250} = \frac{28{,}000}{250} = 112\ \text{mm} \quad \text{O.K.}$$

Incremental deflection

Assuming that time dependent deflection prior to installation of the non-structural elements is negligible, we have

$$(\Delta)_{incr} = (\Delta)_{total} - \left[\Delta_{pi} + (\Delta_i)_{SW} + 0.6 \times (\Delta_i)_{PL}\right] \downarrow$$
$$= 8.3 - (-46.6 + 28 + 0.6 \times 12.5) = 17.8\,\text{mm}$$
$$< \frac{L}{500} = \frac{28{,}000}{500} = 56\ \text{mm}$$
$$< 20\,\text{mm} \quad \text{O.K.}$$

PROBLEM

P5.1 The cross-section of a pre-tensioned concrete beam, simply supported over a span of 28 m, is shown in Fig. P5.1. The beam is to carry a uniformly distributed characteristic dead load of 4 kN/m, in addition to its own weight, and a characteristic imposed load of 10 kN/m, 25% of which should be treated as permanent. The required initial prestressing force of 2610 kN was furnished by stressing 14 numbers of 15.7 mm diameter, 7-wire stands (A_{ps} = 150 mm^2). The eccentricity, e_o, of the prestressing force was maintained constant at 791 mm throughout the length of the beam. The following information is given:

f_{ci} = 30 MPa,	E_{ci} = 26 GPa,	
f_{cu} = 50 MPa,	E_c = 31GPa	
f_{pu} = 1770 MPa,	E_{ps} = 200 GPa	
A_c = 508,000 mm^2;	$I = 134 \times 10^9$ mm^4;	
y_t = 588 mm;	y_b = 912 mm;	η = 0.8

Assuming that 60 percent of the imposed dead load was due to brittle non-structural components installed at the age of 28 days, and that the structure is exposed to an average relative humidity of 75%, check the serviceability limit state of deflection for the beam.

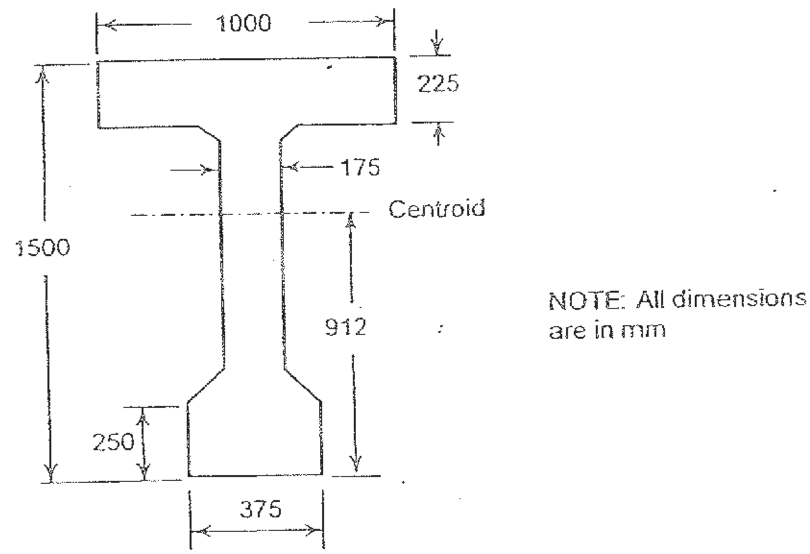


Figure P5.1

Deflections

Short-term deflections

- due to loads on beam and prestress force.

(a) Due to straight tendon

Deflection at mid span $= \dfrac{PeL^2}{8EI}$ ✓

(b) Due to deflected tendon (parabolic)

Deflection at midspan $= \dfrac{L^2}{9.6}\left[\dfrac{1}{r_1} + \dfrac{1}{5}\left(\dfrac{1}{r_2}\right)\right]$ ✓

where : $\dfrac{1}{r_2} = \dfrac{Pe}{EI}$ at support

$\dfrac{1}{r_1} = \dfrac{Pe}{EI}$ at midspan

} upward (↑)

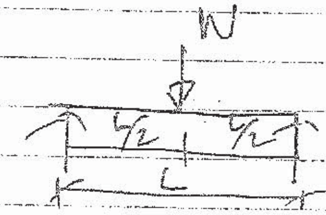
(c) Due to selfweight and UDL Load

Deflection $= \dfrac{5wL^4}{384EI}$ } downward (↓)

Long-term Deflection

Similar calculation as above but use βP and long-term Ec $= \dfrac{E_c}{1+\phi}$ ← short term

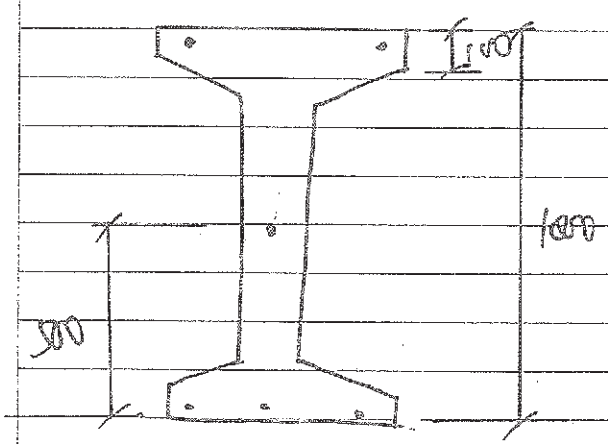
ϕ : creep from cl: 7.3 BS8110: part 2



$\delta = \dfrac{WL^3}{48EI}$

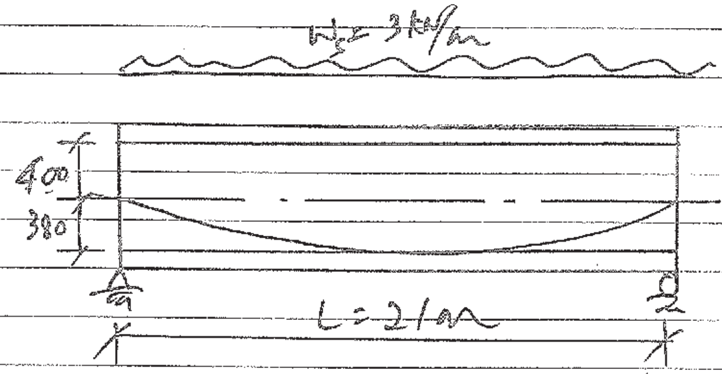
$$(\delta)_{allow} = \frac{L}{360}$$

Example



$E_c = 30\ kN/mm^2$
$A_c = 4 \times 10^5\ mm^2$
$I = 6 \times 10^{10}\ mm^4$
$P = 3300\ kN$
$\gamma_c = 23.6\ kN/m^3$

42 tendon:
- 30 straight tendon below Neutral axis.
- 2 straight tendon above neutral axis
- 10 deflected tendon



(a) Deflection due to Prestress force

(i) Deflection due to straight tendon below Neutral Axis

$$\delta_1 = \frac{P_{30}\, e L^2}{8EI} = \frac{(3300 \times \frac{30}{42}) \times 10^3 \times 380 \times (21 \times 10^3)^2}{8 \times 30 \times 10^3 \times 6 \times 10^{10}}$$

$$= 27.43\ mm\ (\uparrow)$$

(ii) Deflection due to straight tendon above neutral axis

$$\delta_2 = \frac{P_2\, e L^2}{8EI} = \frac{(3300 \times \frac{2}{42}) \times 10^3 \times 400 \times (21 \times 10^3)^2}{8 \times 30 \times 10^3 \times 6 \times 10^{10}}$$

$$= -1.92\ mm\ (\downarrow)$$

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(iii) Deflection due to deflected tendon

$\frac{1}{r_2} = \frac{Pe}{EI} \Rightarrow 0$ ($e \to 0$)

$\frac{1}{r_1} = \frac{Pe}{EI} = \frac{(3300\times\frac{10}{42})\times10^3\times380}{30\times10^3\times6\times10^{10}}$

$= 16.57\times10^{-8}\ mm^{-1}$

$\delta_3 = \frac{L^2}{96}\left[\frac{1}{r_1} + \frac{1}{5}\left(\frac{1}{r_2}\right)\right]$

$= \frac{(21\times10^3)^2}{9.6}\left[16.57\times10^{-8} + 0\right]$

$= 7.61\ mm\ (\uparrow)$

(b) Due to self weight

$W_{sw} = \frac{4\times10^5}{10^6}\times23.6 = 9.44\ kN/m$

$\delta_4 = \frac{5WL^4}{384EI} = \frac{5\times9.44\times(21\times10^3)^4}{384\times30\times10^3\times6\times10^{10}}$

$= 13.24\ mm\ (\downarrow)$

(c) Due to loading

$\delta_5 = \frac{5WL^4}{384EI} = \frac{5\times3\times(21\times10^3)^4}{384\times30\times10^3\times6\times10^{10}}$

$= 4.22\ mm\ (\downarrow)$

Total deflection $= 27.43 - 1.92 + 7.61 - 13.24 - 4.22$

$= 15.66\ mm\ (\uparrow) < \left(\frac{L}{360}\right) = 58.3\ mm$

(OK)