7.1 GENERAL INTRODUCTION

Consider a simply supported I-beam under two symmetrical point loads, as shown in Fig. 7.1(a). The shear force and bending moment diagrams for the beam due to these loads are shown in Figs. 7.1 (b) and (c), respectively. It may be seen that the beam segments AB and CD are subjected to a combined action of bending and shear. In reality, shear is encountered in combination with bending except, for example, at inflection points of a continuous beam.

Fig. 7.1 Stress distribution in an elastic beam section (a) Beam and loading, (b) Shear force diagram, (c) Bending moment diagram, (d) Beam cross section (e) Flexural stress distribution, and (f) Shear stress distribution.
If the beam is assumed to be made up of homogeneous elastic material, then the distribution of stresses at any section X-X within the shear span due to the moment, \( M \) and shear force, \( V \) acting at this section may be obtained from the classical elastic theory. These distributions are shown in Fig. 7.1 (e) and (f), respectively. At any distance \( y \) from the centroidal axis, the normal stress, \( \sigma \) and shear stress, \( \nu \) are respectively given by

\[
\sigma = \frac{My}{I}
\]

and

\[
\nu = \frac{VQ}{lb}
\]

in which \( b \) is the width of the section and \( Q \) is the first moment with respect to the neutral axis of the shaded area shown in Fig. 7.1(d). The state of stress in an infinitesimal element located at this distance is shown in Fig. 7.2(a).

![Figure 7.2 Combined stresses in an element in the tension zone. (a) Stress state, (b) Reduction into bi-axial stress state, (c) Mohr's circle representation of stress state.](image)

The combined stresses (normal and shearing stresses) acting on this element may be resolved into a bi-axial tension-compression stress field [Fig. 7.2(b)]. Using the familiar Mohr’s circle representation, as shown in Fig. 7.2 (c), the principal stresses for the element may be expressed as
Principal tension, 
\[ \sigma_1 = \sqrt{v^2 + \left(\frac{\sigma}{2}\right)^2} + \frac{\sigma}{2} \]  
(7.3)

Principal compression, 
\[ \sigma_2 = \sqrt{v^2 + \left(\frac{\sigma}{2}\right)^2} - \frac{\sigma}{2} \]  
(7.4)

In general, \( V \) and \( M \) vary along the length of the beam (Fig. 7.3), and so do the magnitudes of normal and shearing stresses across the depth of the beam. Hence, the magnitude and direction of principal stresses will vary along the length and across the depth of the beam. Fig. 7.3 shows the principal stress trajectories in a homogeneous isotropic beam subjected to uniform loading.

![Principal stress trajectories for a homogenous isotropic beam.](image)

In the case of structural concrete beams (reinforced or prestressed), the principal stress trajectories would approximately follow the same pattern as long as it remains crack free. When the magnitude of principal tensile stress reaches the tensile strength of the concrete, cracking will occur normal to its direction. As soon as the crack forms, elastic stress distribution near its vicinity will no longer be valid.

### 7.2 CRACKING PATTERNS AND FAILURE MODES

With increasing load beyond first cracking, new cracks will form wherever the principal tensile stress exceeds the tensile strength of the concrete, and the existing cracks will propagate, thus forming a well-developed cracking pattern at ultimate load. The cracking pattern for the beam in Fig. 7.1 under overload condition is shown in Fig. 7.4. Most commonly, three different types of cracks, labelled as Type 1, Type 2 and Type 3 cracks in Fig. 7.4, can be identified when the beam approaches its ultimate load capacity.
Cracks labelled as Type 1 are flexural cracks. These cracks are vertical and occur at locations where bending moment is predominant. Type 2 cracks are known as flexural-shear cracks, that is, flexural (vertical) cracks are formed first, and with further increase in load, they penetrate the beam and become inclined due to the effect of larger shear stress. Type 3 cracks develop within the web where shear force is predominant, but bending moment is small. These cracks are known as web-shear cracks. The beam finally fails due to widening of any of these cracks.

Flexural failure usually occurs by widening of the one of the vertical cracks followed by crushing of the concrete in the compression zone. The strength design for this type of failure has already been treated in Chapter 6. When the failure occurs through an inclined crack, it is known as shear failure, and this type of failure forms the subject matter of the current chapter.

Shear failure may take different forms, as illustrated in Figs. 7.5. The types of failure shown in Figs. 7.5(a), (b) and (c) are respectively known as shear-compression failure, shear-tension failure and diagonal tension failure. The detailed descriptions of these types of failure can be found in related literature. The main point to emphasize here is that web steel yields when failure occurs through an inclined (diagonal) crack. If the beam contains web reinforcement large enough to prevent widening of an inclined crack, failure will occur by crushing of the concrete in between the diagonal cracks, as shown in Fig. 7.5 (d). This type of failure is known as web-crushing failure.
7.3 BENEFITS OF PRESTRESSING IN SHEAR

The above discussion on the cracking pattern and modes of failure applies equally to both reinforced and prestressed concrete beams. The only difference is in the inclination of diagonal cracks. Due to the effect of pre-compression, prestressed concrete beams generally exhibit smaller crack inclination with the horizontal. However, prestressing has two definite advantages over reinforced concrete.

Firstly, a pre-compression (prestress) applied to the concrete would result in a reduction in the principal tensile stress as compared to a corresponding non-prestressed beam. This can be clearly visualised if the element in Fig. 7.2 is considered with a prestress of $\sigma_p$, applied in the longitudinal direction and the corresponding Mohr’s circle is drawn, as in Fig. 7.6. The reduction in principal tension would have the beneficial effect of delaying the formation of cracks in a prestressed concrete beam.

![Figure 7.6 Effect of pre-compression on principal tensile stress. (a) Element with pre-compression, (b) Mohr’s circle showing reduction of principal tension.](image)

The second beneficial effect of prestressing occurs where deflected or draped tendons are used. The effect of prestressing such a tendon may be replaced by a set of equivalent loads acting on the concrete as discussed in Chapter 2. It may be seen in Fig. 7.7 that the equivalent load acting within the span balances part of the external loads. Thus, the net shear to be resisted by concrete is reduced significantly when compared to a corresponding beam without any prestress. It is for this reason that prestressed concrete members are usually made up of thin web sections.

![Figure 7.7 Benefits of deflected tendons.](image)
7.4 ULTIMATE SHEAR RESISTANCE OF A SECTION

With the basic understanding of the effects of shear and the possible types of shear failure, as discussed in the preceding articles, let us now focus our attention to estimating the shear force at a section that would lead to collapse of the beam. It must be pointed out here that shear in concrete members is a highly complex problem, which is yet to be solved with a general agreement like the flexural strength theory. However, there have been some breakthroughs in recent years. With the developments of softened truss model, variable angle truss model, compression field theory, and plasticity theory in shear, perhaps a general agreement on the design approach may be reached in the foreseeable future, and the codes will rewritten accordingly. In the meantime, let us concentrate on the design approach specified in the current building codes.

The intention of this Basic Module is to discuss the British Code approach. It is based on the assumption that the failure occurs by widening of an inclined crack, which extends at an angle of 45-degrees to the beam axis as shown in Fig. 7.8. This mechanism of shear failure is identical to that assumed for reinforced concrete. Through this crack, the applied shear, \( V \) due to ultimate load is carried by the following components, as shown in Fig. 7.8.

(a) Dowel action of longitudinal bars, \( V_d \)
(b) Aggregate interlock action, \( V_a \)
(c) Shear resistance of the concrete compression zone, \( V_{cr} \)
(d) Shear resisted by web reinforcement in tension, \( V_s \)
(e) Vertical component of the inclined tendon force, \( V_p \).

![Figure 7.8 Transfer of shear through an inclined crack.](image)

As the magnitude of the force associated with each of the first three load-carrying mechanisms is difficult to estimate, they are usually lumped together and denoted by a single empirical term, \( V_{AL} \), for the shear strength contributed by the concrete. Thus, the expression for the total shear resistance reduces to

\[
V_R = V_c + V_s + V_p \tag{7.5}
\]
The sign of $V_c$ depends on whether it acts in the same or opposite direction to that of the external shear.

It can be recalled that for reinforced concrete, $v_c = V_c/(b_c d)$ is obtained from Eq. 7.6

$$v_c = 0.63 \left( \frac{100 A_s}{b_c d} \right)^{\frac{1}{3}} \left( \frac{400}{d} \right)^{\frac{1}{2}} \left( \frac{f_{ct}}{25} \right)^{\frac{1}{3}}$$  \hspace{1cm} (7.6)

in which $\frac{100 A_s}{b_c d} \leq 3$; $\frac{400}{d} \geq 1$; and $25 \text{ MPa} \leq f_{ct} \leq 40 \text{ MPa}$

where $A_s$ is the area of tension reinforcement, $d$ is the effective depth of the section and $b_c$ is the breadth of section or average width the rib below the flange for flanged beams.

In the case of prestressed concrete, $V_c$ is taken as the shear force required to produce an inclined crack, where Eq. (7.6) becomes only a component. The following section is devoted to the calculation of the various components that contribute to the ultimate shear resistance of a prestressed concrete beam.

### 7.4.1 Shear at Inclined Cracking, $V_c$

As discussed in Article 7.2, inclined cracks may be classified into two categories - web-shear cracks and flexural-shear cracks. Web shear cracks form in a region uncrazed in flexure, whereas flexural-shear cracks are extensions of flexural cracks. For a particular section, it is therefore necessary to investigate the possibilities of both types of crack formation.

#### Shear at Web-Shear Cracking

Designated as $V_{co}$, the calculation of the shear force that causes web-shear cracks to form is based on rational theory. Since the section remains uncrazed in flexure before the diagonal cracks form, elastic theory may be conveniently applied to calculate the principal tensile stress at c.g.c., where shear stress is maximum and normal stress is the average compressive stress induced by prestressing. This principal tensile stress can then be equated with the estimated tensile strength of the concrete, $f_t$, to compute $V_{co}$.

Consider a prestressed concrete beam section subjected to an eccentric prestress, $F$, as shown Fig. 7.9(a). At this section, the ultimate load acting on the beam produces a bending moment, $M$ and a shear force $V$. The distribution of normal and shearing stresses at this section due to $F$, $M$ and $V$ are also shown in Fig. 7.9(a). The state of stresses for an element taken at the centroid of this section is shown in Fig. 7.9(b), which is represented by Mohr’s circle in Fig. 7.9(c). Obviously, the radius of the circle $R$ is given by

$$R = \sqrt{\left( \frac{\sigma_{co}}{2} \right)^2 + v^2}$$  \hspace{1cm} (7.7)
and the principal tensile stress, \( \sigma_1 \) may be obtained as

\[
\sigma_1 = R - \frac{\sigma_{cp}}{2}
\]  
(7.8)

Figure 7.9 Calculation of principal stresses for a prestressed concrete beam.

Substituting the value of \( R \) from Eq. (7.7) into Eq. (7.8), we obtain \( \sigma_1 \) as a function of the normal stress, \( \sigma_{cp} \) and shearing stress, \( \nu \) as

\[
\sigma_1 = R - \frac{\sigma_{cp}}{2} = \sqrt{\left(\frac{\sigma_{cp}}{2}\right)^2 + \nu^2} - \frac{\sigma_{cp}}{2}
\]  
(7.9)

For a rectangular section of breadth \( b_v \) and overall depth \( D \), the shear stress at the centroidal axis is given by

\[
\nu = \frac{3}{2} \frac{V}{b_v D}
\]  
(7.10)
Substitution of \( v \) from Eq. (7.10) into Eq. (7.9) gives the expression for the applied shear \( V \) as

\[
V = 0.67b_vD\sqrt{\frac{t_v^2}{(1+\sigma_1)\sigma_\infty}}
\]  
(7.11)

Cracking will occur normal to the direction of the principal tensile stress, \( \sigma_1 \) when its value reaches the tensile strength of the concrete, \( f_t \). According to BS 8110-95, tensile strength of the concrete is given by

\[
f_t = 0.24\sqrt{\alpha_w}
\]  
(7.12)

The Code also specifies a partial safety factor of 0.8 for \( \sigma_\infty \). Using this safety factor and replacing \( \sigma_1 \) by \( f_t \) as given by Eq. (7.12) in Eq. (7.11), we obtain the shear force \( V_\infty \), which would cause web-shear cracking as

\[
V_\infty = 0.67b_vD\sqrt{(f_t^2 + 0.8f_t\sigma_\infty)}
\]  
(7.13)

For flanged sections, \( b_v \) is taken as the breadth of the rib as shown in Fig. 7.10. In the case of gradually decreasing web width, the average width of the web below the flange may be used. Where a duct occurs in the rib, the value of \( b_v \) should be reduced by the size of the duct, \( d_a \), if left ungrouted, and two-thirds of its size if grouted subsequently.

(a) Beams without any duct in the rib

(b) Beams containing ducts in the rib

Figure 7.10 Determination of \( b_v \) for flanged beams.
Shear at Flexural-Shear Cracking

The shear force required to transform a flexural crack into an inclined crack may be taken as the sum of the shear force that exists when the flexural crack first develops and the additional shear force required for the flexural cracks ultimately developing into inclined shear cracks. This is denoted by $V_{cr}$. According to BS8110-97, the design equation to estimate $V_{cr}$ is given by the following semi-empirical equation:

$$V_{cr} = \left(1 - 0.55 \frac{a_{pl}}{f_{pu}}\right) V_{c} b_v d + \frac{M_o V}{M} \geq 0.1 b_v d \sqrt{f_{cu}} \quad (7.14)$$

in which $V$ and $M$ are the shear force and bending moment values at the section, respectively, due to ultimate load, and $M_o$ is the moment required to produce zero stress in the concrete at the extreme tension fibre. The value of $M_o$ is given by

$$M_o = 0.8 \sigma_{pl} \frac{l}{y} \quad (7.15)$$

where $\sigma_{pl}$ is the stress in concrete due to prestress alone at the extreme tension fibre, and $y$ is its distance from the c.g.c. Note that a safety factor of 0.8 is applied to the stress $\sigma_{pl}$. In Eq. (7.14), $V_c$ is obtained from the same expression as that for ordinary reinforced concrete by replacing $A_s$ by $(A_s + A_{ps})$ as

$$V_c = 0.63 \left(\frac{100(A_{ps} + A_s)}{b_v d}\right)^{\frac{1}{3}} \left(\frac{400}{f_{cu}}\right)^{\frac{1}{4}} \left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}} \quad (7.16)$$

in which

$$\frac{100(A_{ps} + A_s)}{b_v d} \leq 3; \quad \frac{400}{d} \geq 1; \quad \text{and} \quad 25 \text{MPa} \leq f_{cu} \leq 40 \text{MPa}$$

The Value of $V_c$

To evaluate the design ultimate shear resistance of the concrete, $V_c$ for a given section along the length of the beam, it is first necessary to calculate the value of $M_o$ using Eq. (7.15). If $M < M_o$, then the section is considered as uncracked, otherwise it is regarded as cracked in flexure. According to the code,

$$V_c = V_{cc}$ at uncracked sections ($M < M_o$), and
$$V_c = \text{smaller of} \{V_{cc} \text{ and } V_c\}$ at cracked sections ($M \geq M_o$). \quad (7.17a)$$

(7.17b)

2.4.2 Contribution of Shear Reinforcement, $V_s$

Let us consider the general case of inclined stirrups, as shown in Fig. 7.11. It is obvious that the contribution of web reinforcement, $V_s$, to the shear strength of the beam
depends on its inclination with the horizontal, \( \alpha_v \), the force, \( F_v \), carried by the vertical legs of each stirrup, the spacing of stirrups, \( s \), and the slope of the inclined failure crack, \( \theta \). It is generally assumed that stirrups yield at failure. Therefore, inserting the partial safety factor, \( \gamma_m \) for steel, \( F_v = A_{sv} f_{yv} \gamma_m \). Also, in the BS Code, the angle, \( \theta \), is conservatively assumed as 45-degrees. With these assumptions, the shear strength attributable to reinforcement can be easily derived as

\[
V_s = \frac{A_{sv} f_{yv} (\sin \alpha_v + \cos \alpha_v)}{\gamma_m} \frac{d_i}{s}
\]  

(7.18)

in which \( d_i \) is the distance from the extreme compression fibre to the centroid of the outermost layer of tensile steel.

Figure 7.11 Shear strength \( V_s \) provided by shear reinforcement.

For vertical stirrups, which are commonly used in prestressed concrete construction, \( \alpha_v = 90^\circ \). Eq. (7.18) then reduces to

\[
V_s = \frac{1}{\gamma_m} \frac{A_{sv} f_{yv} d_i}{s}
\]

(7.19)

7.4.3 Vertical Component of Prestressing, \( V_p \)

The vertical component, \( V_p \) of the prestressing force at a particular section is simply the magnitude of the prestressing force, \( F_p \), times \( \sin \alpha \), where \( \alpha \) is the inclination of the tendon at the section under consideration, that is

\[
V_p = F_p \sin \alpha
\]

(7.20)

7.4 Design of Shear Reinforcement at a Section

The design requirement for a given section may be stated as follows:

\[
V \leq V_R
\]

(7.21)
that is, the shear force, $V$, at the section due to ultimate loads should be less than or equal to the shear resistance, $V_R$, provided by the section. At equality, using Eq. (7.5), Eq (7.21) may therefore be written as

$$V = V_c + V_s + V_p$$  

(7.22)

For the purpose of design, the effect of $V_p$ may be thought of as an external shear, which either increases or decreases the applied shear, $V$ depending on its direction. In other words, $V$ and $V_p$ in Eq. (7.22) may be lumped together algebraically and denoted by $V_D$. That is,

$$V_D = V - V_p$$  

(7.23)

where $V$ and $V_p$ are in the opposite directions.

According to BS 8110-1997, the effect of $V_p$ should always be considered if it increases the external design load effects. Under the commonly encountered cases where $V_p$ acts in a direction opposite to the external shear, the beneficial effect of prestressing is however ignored when the section is cracked in flexure.

Inserting the value of $V_s$ from Eq. (7.19) and denoting $V_D = V - V_p$, Eq. (7.23) becomes

$$V_D = V_c + \frac{1}{Y_m} \frac{A_{sv} f_{sv}}{s} d_t$$  

(7.24)

Reorganising Eq (7.24), we obtain the design equation as

$$A_{sv} = \frac{V_D - V_c}{f_{sv}} \frac{d_t}{s}$$  

(7.25)

### 7.5 LIMITATIONS ON SHEAR REINFORCEMENT

**Minimum shear reinforcement**

In order to avoid a sudden and explosive failure under accidental overloading, British code requires that a minimum amount of shear reinforcement be provided whenever $V_D$ exceeds $0.5 V_c$, but remains within $V_c + 0.4 b_s d$. This amount is given by

$$\begin{bmatrix} A_{sv} \\ s_v \end{bmatrix}_{min} = \gamma_m \left( \frac{0.4b_s}{f_{sv}} \right)$$  

(7.26)

**Maximum design shear force**

In a beam with large amount of web reinforcement, failure may occur by crushing of the concrete in between the inclined cracks before yielding of the web steel. This type of premature failure is referred to as web crushing failure. In design, web crushing failure
is avoided by setting an upper limit to the shear force generated by design ultimate load. That is,

\[ V_D \]_{\text{max}} = 0.8 \sqrt{f_{cu}} b_y d \quad \text{or} \quad 5b_y d \quad (7.27)

whichever is lesser.

7.6 CONSIDERATIONS FOR DETAILING

7.6.1 Types of Web Reinforcement

Shear reinforcement in prestressed concrete beams consists mostly of vertical links. Typical links are shown in Fig. 7.12. They can be either open at the top or closed as shown, but must extend the full depth of the section, enclose all the tendons and additional reinforcement provided at the section and be adequately anchored.

![Typical shear reinforcement](image)

Figure 7.12 Typical shear reinforcement.

7.6.2 Anchorage of Web Reinforcement

For adequate anchorage, each corner of a link must contain a longitudinal bar, a tendon or a group of tendons whose diameter should not be smaller than the diameter of link bar itself. At the open ends of a link, anchorage can be accomplished by providing a bend, a hook or welded transverse wires, the requirements of which are illustrated in Fig. 7.13.

![Anchorage of links](image)

Figure 7.13 Anchorage of links.
7.6.3 Maximum Spacing of Stirrups

For shear reinforcement to function properly without any localised failure before the
load-carrying mechanism is fully established, some limitations on maximum spacing of
stirrups must be followed. British code (1997) limits the stirrup spacing as follows:

When \( V_D < 1.8 V_c \)
\[
 s_v \leq 0.75d_i \quad \text{or} \quad s_v \leq 4b_i \quad \text{(for flanged sections)} \tag{7.28a}
\]

When \( V_D \geq 1.8 V_c \)
\[
 s_v \leq 0.5d_i \tag{7.28b}
\]

**EXAMPLE 7.1**

A prestressed concrete T-beam [Fig. E7.1.1(a)], simply supported over a span of 28
m, has been designed for flexure to carry, in addition to its own weight, a
characteristic imposed dead load \( 4 \) kN/m\(^2\) and a characteristic imposed live load of
10 kN/m. The beam is pre-tensioned with 14-15.7 mm diameter super strands, each
with nominal cross-sectional area, \( A_{ps} = 150 \) mm\(^2\), but due to debonding only 7 of the
strands, as shown in Fig. E7.1.1(b) by filled circles are active at a section 2 m from
the support. These seven strands supply an effective prestressing force of 1044 kN.

The following information is given:

\[
 A_c = 508,000 \text{ mm}^2; \quad I = 134 \times 10^9 \text{ mm}^4; \quad y_D = 912 \text{ mm}
\]

\[
 f_{cu} = 50 \text{ MPa}; \quad f_{pu} = 1770 \text{ MPa}; \quad f_{yv} = 250 \text{ MPa}
\]

Design the section at a distance of 2 m from the support for shear.

![Figure E7.1.1](image-url)
SOLUTION

(a) Loads

Self-weight
\[ w_c = 0.508 \times 24 = 12.19 \text{ kN/m} \]

Ultimate load
\[ w_u = (12.19 + 4) \times 1.4 + 10 \times 1.6 = 38.67 \text{ kN/m} \]

(b) Shear and moment at the section due to ultimate load

At the section, 2 m from the support
\[ V = 38.67 \left( \frac{28}{2} - 2 \right) = 464 \text{ kN} \]
\[ M = 38.67 \times \frac{28}{2} \times 2 - 38.67 \times \frac{2^2}{2} = 1005 \text{ kNm} \]

(c) Calculate \( M_a \)

Distance, \( y \), of the centroid of active tendons from the bottom face of the section
\[ y = \frac{5 \times 62.6 + 2 \times 3 \times 62.5}{7} = 98 \text{ mm} \]

Therefore,
\[ e_o = 912 - 98 = 814 \text{ mm} \]
\[ d = 1500 - 98 = 1402 \text{ mm} \]

Stress in concrete at the extreme tension fibre due to prestress alone is
\[ \sigma_{pt} = \frac{F_p}{A_c} = \frac{F_p}{l} = \frac{1044 \times 10^3 + 1044 \times 10^3 \times 814 \times 912}{508,000 + 134 \times 10^8} = 7.84 \text{ MPa} \]

Therefore, moment, \( M_a \) is obtained as
\[ M_a = 0.8 \sigma_{pt} \frac{l}{y} = 0.8 \times \frac{7.84 \times 134 \times 10^3 \times 10^{-6}}{912} = 922 \text{ kNm} \]
\[ < M = 1005 \text{ kNm} \]

Since \( M > M_a \), the section is cracked in flexure, and it is therefore necessary to evaluate both \( V_{cp} \) and \( V_{ct} \) to find \( V_c \).
(d) Calculation of $V_{co}$

\[ \sigma_{co} = \frac{F_e}{A_c} = \frac{1044 \times 10^3}{508,000} = 2.06 \text{ MPa} \]

\[ f_c = 0.24 \sqrt{f_{co}} = 0.24 \sqrt{50} = 1.70 \text{ MPa} \]

With $h = 1500$ mm and $b_v = 175$ mm, we obtain the shear force at web-shear cracking as:

\[ V_{co} = 0.67 b_v D \sqrt{\left( f_c^2 + 0.8 f_c \sigma_{cp} \right)} \]
\[ = 0.67 \times 175 \times 1500 \times 10^{-3} \sqrt{1.70^2 + 0.8 \times 1.7 \times 2.06} = 420 \text{ kN} \]

(e) Calculation of $V_{cr}$

\[ \sigma_{ps} = \frac{F_e}{A_{ps}} = \frac{1044 \times 10^3}{7 \times 150} = 994 \text{ MPa} \]

**Calculation of $v_c$**

\[ \frac{100 (A_{ps} + A_s)}{b_v d} = \frac{100 \times 7 \times 150}{175 \times 1402} = 0.43 \leq 3 \ \text{O.K.} \]

\[ \frac{400}{d} = \frac{400}{1402} = 0.29 < 1; \quad \text{Use } \frac{400}{d} = 1 \]

\[ f_{cu} = 50 \text{ MPa} > 40 \text{ MPa}; \quad \text{Use } f_{cu} = 40 \text{ MPa} \]

Therefore,

\[ v_c = 0.63 \left( \frac{100 (A_{ps} + A_s)}{b_v d} \right)^{\frac{1}{3}} \left( \frac{400}{d} \right)^{\frac{1}{4}} \left( \frac{f_{cu}}{25} \right)^{\frac{1}{3}} \]
\[ = 0.63 \times 0.43^{\frac{1}{3}} \times 1 \times \left( \frac{40}{25} \right)^{\frac{1}{3}} = 0.556 \text{ MPa} \]

Thus, the shear force, $V_{cr}$, at flexural-shear cracking is

\[ V_{cr} = \left( 1 - 0.55 \frac{\sigma_{ps}}{f_{ps}} \right) v_c b_v d + \frac{M_s Y}{L} \geq 0.1 b_v d \sqrt{f_{cu}} \]
\[ = \left( 1 - 0.55 \times \frac{994}{1770} \right) \times 0.556 \times 175 \times 1402 \times 10^{-3} + 922 \times \frac{464}{1005} = 520 \text{ kN} \]
\[ > 0.1 \times 175 \times 1402 \sqrt{50} \times 10^{-3} = 174 \text{ kN} \ \text{O.K.} \]

(f) Shear resistance provided by the concrete, $V_c$

\[ V_c = \text{smaller of } \{ V_{co} \text{ and } V_{cr} \} \]
\[ = \text{smaller of } \{ 420 \text{ and } 520 \text{ kN} \} = 420 \text{ kN} \]
(g) Design of shear reinforcement

\[ V_D = V = 464 \text{ kN} \]
\[ > 0.5V_c = 0.5 \times 420 = 210 \text{ kN} \]
\[ < V_c = 0.4b_y d = 518 \text{ kN} \]

Therefore, only nominal links are required. Assuming \( \gamma_m = 1.15 \), it is obtained as follows:

\[ \frac{A_{sv}}{s_v} = \gamma_m \left( \frac{0.4b_y}{f_{yw}} \right) = 1.15 \times \frac{0.4 \times 175}{250} = 0.322 \text{ mm}^2/\text{mm} \]

Using 10 mm diameter U-links, the required spacing is

\[ s_v = \frac{2 \times 78.5}{0.322} = 487 \text{ mm} < 0.75d_i = 0.75 \times (1500 - 62.5) = 1078 \text{ mm} \]
\[ < 4b_y = 4 \times 175 = 700 \text{ mm} \]

Use 10 mm diameter U-links at 450 mm c/c.

The shape and arrangement of the links are shown in Fig. E7.1.2.

![Diagram of links](image)

Fig. E7.1.2 Shape and arrangement of the links.

Looking back at the arrangement of strands at midspan in Fig. E7.1.1, it may be seen that the chosen shape and arrangement of links shown in Fig. 7.1.2 cannot be accommodated because the requirement of clear concrete cover for the web will then be violated. Therefore, while deciding on the arrangement of strands, the designer should always keep in view the probable shape and arrangement of links that will eventually be required for carrying transverse shear in order to avoid possible design revision.
For the problem at hand, some revision of tendon arrangement is clearly needed. One possible option for such a revision is shown in Fig. 7.1.3.

![Revised arrangement of strands](image)

Figure E7.1.3 Revised arrangement of strands.

With such a change in the arrangement of strands, it is necessary to go back and check its effects on the previous design steps.

**EXAMPLE 7.2**

A post-tensioned concrete T-beam, simply supported over a span of 28 m has been designed for flexure to carry, in addition to its own weight, a characteristic imposed dead load of 4 kN/m² and a characteristic imposed live load of 10 kN/m. The selected cable profile for the beam is parabolic with eccentricities of zero at each end and 790 mm at midspan as shown in Fig. E7.2.

![Beam in Example 7.2](image)

Figure E7.2 Beam in Example 7.2
The tendons are contained in two ducts, each 60 mm in diameter. They are arranged in a vertical plane through the web and are grouted later. Four T16 bars were provided near the bottom face of the beam, as shown, for anchorage of stirrups. The following information is given:

\[ A_c = 508,000 \text{ mm}^2; \quad I = 134 \times 10^9 \text{ mm}^4; \quad y_b = 912 \text{ mm} \]
\[ f_{cu} = 50 \text{ MPa}; \quad f_{ps} = 1770 \text{ MPa}; \quad f_{ty} = 250 \text{ MPa} \]
\[ A_{ps} = 2100 \text{ mm}^2; \quad F_{pe} = 2088 \text{ kN} \]

Design the section 2 m away from the support for shear.

**SOLUTION**

(a) Loads

Self-weight

\[ w_0 = 0.508 \times 24 = 12.19 \text{ kN/m} \]

Ultimate load

\[ w_u = (12.19 + 4) \times 1.4 + 10 \times 1.6 = 38.67 \text{ kN/m} \]

(b) Shear and moment at the section due to ultimate load

At the section, 2 m from the support

\[ V = 38.67 \left( \frac{28}{2} - 2 \right) = 464 \text{ kN} \]

\[ M = 38.67 \times \frac{28}{2} x - 38.67 \times \frac{2^2}{2} = 1005 \text{ kNm} \]

(c) Calculate \( M_o \)

Eccentricity of c.g.s. at \( x = 2 \text{ m} \)

\[ e_o = \frac{4 \delta}{L^2} x^2 + \frac{4 \delta}{L} x = -\frac{4 \times 790 \times 2^2}{28^2} + \frac{4 \times 790 \times 2}{28} = 210 \text{ mm} \]

Therefore,

\[ \sigma_{pl} = \frac{F_{e} + F_{e} \cdot y \cdot y_b}{A_c} \]
\[ = \frac{2088 \times 10^3}{508,000} + \frac{2088 \times 10^3 \times 210 \times 912}{134 \times 10^9} = 7.09 \text{ MPa} \]

\[ M_o = 0.8 \sigma_{pl} \frac{I}{y} = 0.8 \times \frac{7.09 \times 134 \times 10^9}{912} \times 10^{-6} = 833 \text{ kNm} < M = 1005 \text{ kNm} \]

Since \( M > M_o \), the section is cracked in flexure, and it is therefore necessary to evaluate both \( V_{co} \) and \( V_{cr} \) to find \( V_c \).
(d) Calculation of $V_{co}$

$$\sigma_{co} = \frac{F_e}{A_e} = \frac{2088 \times 10^3}{508,000} = 4.11 \text{MPa}$$

$$f_t = 0.24 \sqrt{f_{cm}} = 0.24 \sqrt{50} = 1.70 \text{MPa}$$

$$b_v = \frac{2}{3} \times 60 = 135 \text{mm}$$

With $h = 1500 \text{mm}$, we obtain the shear force at web-shear cracking as follows:

$$V_{co} = 0.67 b_v D \sqrt{\left[f_t^2 + 0.8 f_c \sigma_{co}\right]}$$

$$= 0.67 \times 135 \times 1500 \times 10^{-3} \sqrt{1.70^2 + 0.8 \times 1.7 \times 4.11} \approx 395 \text{kN}$$

(e) Calculation of $V_{cr}$

$$\sigma_{pe} = \frac{F_e}{A_{ps}} = \frac{2088 \times 10^3}{2100} = 994 \text{MPa}$$

Calculate $d_p$

$$d_p = y_1 + e_o = (1500 - 912) + 210 = 798 \text{mm}$$

Therefore,

$$d = \frac{A_{ps} d_p + A_s d_s}{A_{ps} + A_s} = \frac{2100 \times 798 + 804 \times 1440}{2100 + 804} = 976 \text{mm}$$

Calculate $v_c$

Check the limits

$$\frac{100(A_{ps} + A_s)}{b_v d} = \frac{100 \times (2100 + 804)}{135 \times 976} = 2.21 \leq 3 \text{ O.K.}$$

$$\frac{400}{d} = \frac{400}{976} = 0.41 < 1; \quad \text{Use } \frac{400}{d} = 1$$

$$f_{cr} = 50 \text{ MPa} > 40 \text{ MPa}; \quad \text{Use } f_{cr} = 40 \text{ MPa}$$

Therefore,

$$v_c = 0.63 \left(\frac{100(A_{ps} + A_s)}{b_v d}\right)^{\frac{1}{3}} \left(\frac{400}{d}\right)^{\frac{1}{4}} \left(\frac{f_{cr}}{25}\right)^{\frac{1}{5}}$$

$$= 0.63 \times 2.21^{\frac{1}{3}} \times 1 \times \left(\frac{40}{25}\right)^{\frac{1}{5}} = 0.96 \text{ MPa}$$
Thus, the shear force, $V_c$, at flexural-shear cracking is

$$V_c = \left(1 - 0.55 \frac{f_{cu}}{f_{cu}}\right) v_d b_y d + \frac{M_o V}{M} \geq 0.15 b_v d \sqrt{f_{cu}}$$

$$= \left(1 - 0.55 \times \frac{994}{1770}\right) \times 0.96 \times 135 \times 976 \times 10^{-3} + 833 \times \frac{464}{1005} = 472 \text{ kN}$$

$$> 0.1 \times 135 \times 976 \sqrt{50} \times 10^{-3} = 93 \text{ kN} \quad \text{O.K.}$$

(f) Shear resistance, $V_c$, provided by the concrete,

$$V_c = \text{smaller of } \{V_{cu} \text{ and } V_{cr}\}$$

$$= \text{smaller of } \{395 \text{ and } 472 \text{ kN}\} = 395 \text{ kN}$$

(g) Design of shear reinforcement

*Calculation of $V_p$*

Differentiating the expression for the cable profile with respect to $x$, and putting $x = 2 \text{ m}$ in the resulting expression, we obtain the slope of the tendon as

$$\theta |_{x=2} = \left(\frac{d e_o}{d x}\right) |_{x=2} = \frac{8\delta}{L^2} x + \frac{4\delta}{L} = -\frac{8 \times 0.790 \times 2}{28^2} + \frac{4 \times 0.790}{28} = 0.097 \text{ rad.}$$

Therefore,

$$V_p = 0.097 \times 2088 = 202 \text{ kN} \quad \text{(beneficial)}$$

Since $M > M_o$, the section is considered cracked in flexure. For a cracked section, the beneficial effect of prestressing is ignored, $V_p$ is taken as zero.

Hence

$$V_D = V - V_p = 464 - 0 = 464 \text{ kN}$$

$$> 0.5V_c = 0.5 \times 395 = 198 \text{ kN}$$

$$> V_c + 0.4b_v d = 448 \text{ kN}$$

Therefore, shear reinforcement is required as

$$\frac{A_{as}}{s_v} = \left(\frac{V_D - V_c}{0.85f_{y,y} d_1}\right) \left(\frac{464 - 395}{0.67 \times 250 \times 1440}\right) \times 10^3 = 0.22 \text{ mm}^2/\text{mm}$$

Using 10 mm diameter U-links, the require spacing is

$$s_v = \frac{2 \times 78.5}{0.22} = 713 \text{ mm}$$

$$< 0.75d_1 = 0.75 \times (1500 - 60) = 1080 \text{ mm}$$

$$> 4b_v = 4 \times 135 = 540 \text{ mm}$$

*Use 10 mmφ U-links at 500 mm c/c.*
7.7 DESIGN OF BEAMS FOR SHEAR

The preceding sections describe in detail the design of a particular section for shear. For designing the entire beam, it is necessary to consider several sections along its length. The requirement of shear reinforcement at each of these discrete sections can then be calculated. The design for the entire beam may then be obtained simply by assembling the information thus generated. While detailing, the designer should try to employ the same size and shape of the links, at least for that particular beam, and try not to use too many variations in the spacing of stirrups so as to avoid possible construction error.

In contrast to reinforced concrete where the critical section usually occurs near the support, in prestressed concrete several sections along the length of the beam may become equally or more critical than the support section. This may be illustrated by plotting the variation of both the design shear, $V_D$ and the shear resistance, $V_c$ provided by the concrete along the span of a uniformly loaded beam, as shown in Fig. 7.14. The difference between $V_D$ and $V_c$, as represented by the dotted area, denotes the design shear strength that must be provided by the shear reinforcement.

It may be seen in the qualitative representation of shear reinforcement requirement in Fig. 7.14 that there are two critical sections, one close to the support and the other is within the span, where the amount of shear reinforcement required is higher than the critical section adjacent to the support.

![Figure 7.14 Requirements of shear reinforcement for a uniformly loaded beam.](image)

The first critical section for shear is usually taken at a distance $D/2$ for post-tensioned beams and $y_S$ for pre-tensioned beams from the face of the support or bearing, as shown in Fig. 7.14, and the same shear reinforcement is continued back into the support.

For the design of an entire beam, a number of sections, say at an interval of 0.1 $L$, are selected in addition to the first critical section, and the reinforcement
requirement for each section is estimated as in Examples 7.1 or 7.2. Assembling the
designs of these discrete sections, the final distribution of links can be easily arrived at. For the sake of construction simplicity, usually three to four constant spacing is
selected to approximate the continuously varying reinforcement requirements along
the length of the beam.

**Problem 7.1**

The post-tensioned concrete beam of Fig. P7.1 is prestressed by a bonded cable \( A_{pa} = 2,400 \, \text{mm}^2 \) with an eccentricity of 700 mm at midspan and zero at each support. The
effective prestressing force is 2,600 kN at each support and 2,500 kN at midspan, and
is assumed to vary linearly along the beam length. Two 16mm-diameter ordinary
rebars are provided near the tension face, as shown for anchorage of stirrups. The
following information is given:

- Characteristic imposed dead load = 10 kN/m
- Characteristic imposed live load = 20 kN/m
- \( A_c = 570,000 \, \text{mm}^2 \)
- \( f_{cu} = 40 \, \text{MPa} \)
- \( f_{iy} = 250 \, \text{MPa} \)
- \( I = 96.3 \times 10^6 \, \text{mm}^4 \)

If \( c_{pe}/f_{p0} = 0.56 \) for the section 2.5 m from the support,

(a) determine the spacing of 12 mm diameter links at the section.

(b) sketch the section showing the arrangement of links.

![Figure P7.1](image-url)
PROBLEM 7.2

A precast, pretensioned concrete beam (Fig. P7.2), simply supported over a span of 10 m has been designed for flexure to carry, in addition to its own weight, a characteristic imposed dead load 1.5 kN/m² and a characteristic imposed live load of 3 kN/m². The beam is pretensioned with 4-12.9 mm diameter strands, two in each rib, but due to debonding only 2 of the strands are active at a section 1 m from the support. These two strands supply an effective prestressing force of 208 kN. The following additional information is given:

\[ A_0 = 134,000 \text{ mm}^2; \quad l = 1.115 \times 10^5 \text{ mm}^4; \quad y_b = 200 \text{ mm} \]
\[ f_{cu} = 50 \text{ MPa}; \quad f_{yr} = 250 \text{ MPa}; \quad d_r = 275 \text{ mm} \]

(a) Design the section at a distance of 1 m from the support for shear.

(b) Sketch the arrangement of links at the section.

![Figure P7.2](image)

PROBLEM 7.3

A prestressed concrete girder, simply supported over a span of 20 m has been designed for flexure to carry, in addition to its own weight, a characteristic imposed dead load of 4.5 kN/m and a characteristic imposed live load of 10 kN/m. The beam is pretensioned with 16 – 15.7 mm diameter strands (nominal cross-sectional area of each strand, \(A_{ps} = 150 \text{ mm}^2\)), but due to debonding only 8 of the strands are active at a section 3 m from the support (Fig. P7.3). These eight strands supply an effective prestressing force of 1200 kN. The following additional information is given:

\[ A_0 = 324,000 \text{ mm}^2; \quad l = 38.1 \times 10^9 \text{ mm}^4; \quad y_b = 445 \text{ mm} \]
\[ f_{cu} = 40 \text{ MPa}; \quad f_{yr} = 250 \text{ MPa}; \quad \sigma_{pe}/f_{pu} = 0.56 \]

If the section is cracked in flexure, calculate \(V_{cr}\) for the section and determine the spacing of 10 mm diameter, two-legged links required at the section.
All dimensions are in mm

Figure P7.3
Design for shear

Zone A
- Zone extends to a distance of from the support
- Crack-free

Zone B
- Shear cracks develop within the web
  - Web shear cracks

Zone C
- High bending moment
  - Vertical flexural cracks

Zone D
- Bending moment is dominant and shear is insignificant
- Only vertical flexural cracks are formed and shear failure can be ruled out
Zone A & B combined as the zone of web-shear cracking.

Zone C as flexural-shear cracking.

In BS 8110, the shear-carrying capacity of the zone A-B is designated as $V_{co}$. Zone C is designated as $V_{cr}$.

Uncracked sections (c 7.2.3.8-4)

$$V_{co} = 0.671h (f_t + 0.8f_{tp} f_x) y$$

Where:

$$f_t = 0.24 f_{cm}$$

$$f_{tp} = \frac{f_p}{A_2}$$

Values of $V_{co}/h$ for different concrete grades and levels of prestress are given in Table 4.5.

BS 8110: Part 1

<table>
<thead>
<tr>
<th>$f_{yu}$</th>
<th>Concrete grade</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
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<td>N/mm²</td>
<td>N/mm²</td>
<td>N/mm²</td>
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<td>1.95</td>
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<td></td>
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<td>2.20</td>
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<td>2.85</td>
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<td>2.75</td>
<td>2.95</td>
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<td>2.70</td>
<td>2.70</td>
<td>3.15</td>
<td>3.30</td>
<td></td>
</tr>
</tbody>
</table>
Cracked Actions
- $V_{cr} = (1 - 0.05 \frac{f_{pc}}{f_{pu}}) V_{cd} + \frac{M_o V}{M} = 0.16 V_{fu}$

where: $f_{pc} = f_e \times 0.67 f_{pu}$

$V_{c} = \frac{0.79}{125} \left( \frac{12f_{cd}}{b_d} \right)^{2/3} \left( \frac{800 f_{cd}}{f_{cu}} \right)^{1/3}$

$M_o = 0.8 f_{pc} \left( \frac{V}{g} \right)$

Step of shear design to BS 8110

1. Determine uncracked and cracked zones along the beam.

2. Prepare a table for the effective shear force, $V_{eff}$, along the beam.
### Unchecked

<table>
<thead>
<tr>
<th>X (m)</th>
<th>V (KN)</th>
<th>A (cm²)</th>
<th>P (KN)</th>
<th>V - P</th>
<th>Vf (KN)</th>
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</thead>
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<td>8</td>
</tr>
</tbody>
</table>

### Diagram

Unracked section: \( V_f = V - P \)

Cracked section: \( V_f = \) the greater of \( V - P \) or \( V \)

2. Prepare a table for the effective shear resistance, \( V_c \).

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
<th>Column 6</th>
<th>Column 7</th>
<th>Column 8</th>
<th>Column 9</th>
<th>Column 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>X (m)</td>
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<td>c (cm)</td>
<td>M (KNm)</td>
<td>h (cm)</td>
<td>Vf (KN)</td>
<td>V (KN)</td>
<td>Vc (KN)</td>
<td>Vc (KN)</td>
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<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

Unracked section: \( V_c = V_{c0} \)

Cracked section: \( V_c = \) the lesser of \( V_{c0} \) and \( V_f \)
3. Plot $V_{eff}$ and $V_e$.

4. Design shear reinforcement.

Example of plot:

Shear force diagram

<table>
<thead>
<tr>
<th>Designed links</th>
<th>Normal links</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{A_{sv}}{S_v} \geq 0.046$</td>
<td>$\frac{b(V - V_e)}{0.02f_y}$</td>
</tr>
</tbody>
</table>

Diagram showing the relationship between $V_{eff}$, $V_e$, and the shear force, with designated and normal links.
EXAMPLE 7.1

The beam shown in Fig. 7.4 supports an ultimate load, including self weight, of 85 kN/m over a span of 15 m and has a final prestress force of 2000 kN. Determine the shear reinforcement required. Assume that $f_{cu} = 40$ N/mm$^2$.

Section properties: $A_c = 2.9 \times 10^4$ mm$^2$

$I = 3.54 \times 10^{10}$ mm$^4$.

![Fig. 7.4](image)

Fig. 7.5 Bending moment and shear force diagrams for beam in Example 7.1.

The ultimate bending moment and shear force diagrams are shown in Figs 7.5(a) and (b), respectively.

The maximum allowable shear force in the section is given by

$V_{max} = 0.8 \times 40^{1/2} \times 150 \times 1000 \times 10^{-3}$

$= 758.9$ kN.

For the uncracked regions of the beam, since the centroid of the section lies within the web, the shear resistance is given by Equation 7.6 with $f_{cr}$ taken at the centroid

$f_{cr} = 2000 \times 10^9/(2.9 \times 10^8)$

$= 6.90$ N/mm$^2$.

Thus, from Table 7.1, $V_{cr}/bh = 2.19$ N/mm$^2$ and

$V_{cr} = 2.19 \times 1000 \times 150 \times 10^{-3}$

$= 328.5$ kN.

At the supports, the slope of the prestressing tendons is given by

$\theta = \tan^{-1}(4d_i/L)$,

where $d_i$ is the drape of the tendons.
\[ \theta = \tan^{-1}\left(4 \times 425/15000\right) \]
\[ = 6.47^\circ. \]

The vertical component of the shear force at the face of the support is 2000 \sin \theta, or 225.2 kN, so that the total shear resistance at the support is 553.7 kN.

The total uncracked shear resistance will decrease at points further into the beam, assuming a constant effective prestress force along its length, since the angle of inclination of the tendons, \(\theta\), will reduce. The variation of \(V_{te}\) along the length of the beam is shown in Fig. 7.6.

For the cracked regions of the beam, the shear resistance is given by Equation 7.13. The area of the prestressing tendons is 2010 mm\(^2\) and \(f_{pc}/f_{pu}\) is assumed to be constant at 0.6. At 3 m from the support, the depth of the tendons is given by
\[
d = 500 + (4 \times 425/150^2) \times 3(15 - 3) \\
= 772 \text{ mm.}
\]

Thus,
\[
100 A_d/bd = (100 \times 2010)/(150 \times 772) \\
= 1.74.
\]

Therefore, from Table 7.2, \(v_c = 0.76 \text{ N/mm}^2\).

\[
f_m = \frac{P}{A} + \frac{P_{ey}}{I} \\
= \frac{2000 \times 10^3}{2.9 \times 10^3} + \frac{2000 \times 10^3 \times 272 \times 500}{3.54 \times 10^{16}} \\
= 14.58 \text{ N/mm}^2.
\]

\[ \therefore M = (0.8 \times 14.58 \times 3.54 \times 10^6/500) \times 10^{-6} = 825.8 \text{ kN m}. \]

Also,
\[ V = 382.5 \text{ kN}; \quad M = 1530.0 \text{ kN m}. \]

\[ V_{te} = (1 - 0.55 \times 0.6) \times 0.76 \times 150 \times 772 \times 10^{-3} \\
+ (825.8 \times 382.5/1530.0), \]
\[ = 265.4 \text{ kN}. \]

Also,
\[ V_{te} \geq 0.1 \times 150 \times 772 \times 40^{1/2} \times 10^{-3} = 73.2 \text{ kN}. \]

The cracked shear resistance must be checked at other sections along the beam and a plot of this is shown in Fig. 7.6.

It can be seen from Fig. 7.6 that everywhere along the beam, except very near the supports, \(V_{te} < V_{te}^*\) and that, except for a small region near the centre of the beam, \(V > 0.5 \times V_{te}^*\), where \(V_{te}^*\) is the lesser of \(V_{te}\) and \(V_{te}^*\), and thus shear reinforcement is required. The nominal links required are given by
Equation 7.14, i.e.

$$A_{s_v}/s_v = (0.4 \times 150)/(0.87 \times 250)$$

$$= 0.276.$$  

For R8 links at 350 centres, $A_{s_v}/s_v = 0.287$. The total shear resistance of the beam with nominal links throughout is shown in Fig. 7.6, and it can be seen that this shear resistance is adequate for the centre 5 m of the beam, but that everywhere else more substantial reinforcement is required.

The maximum difference between the applied shear force and $V_e$ is approximately 130 kN, and this must be resisted by shear reinforcement, with cross-sectional area and spacing given by Equation 7.16.

That is,

$$A_{s_v}/s_v = (130 \times 10^3)/(0.87 \times 250 \times 772)$$

$$= 0.774.$$  

Thus R12 links at 275 mm centres would be adequate, giving $A_{s_v}/s_v = 0.823$.  

---

*Fig. 7.6 Shear resistance of beam in Example 7.1.*