5.1 GENERAL INTRODUCTION

For the purpose of design, a concrete structure is usually divided into two types of regions, namely, the main regions and the local regions. In the main regions (often denoted as B-regions), stresses and strains are distributed so regularly that they can be easily expressed mathematically. That is, stresses and strains in the main regions are governed by simple equilibrium and compatibility conditions. The design of such a region has so far been treated in this text.

In contrast, in a local region (denoted as D-regions), such as the end of a beam or a column, the beam-column connections, the region adjacent to a concentrated load or a transverse opening, the stresses and strains are so disturbed and irregular that they are not amenable to mathematical formulation using the basic requirements of equilibrium, compatibility and material laws. Here, the compatibility conditions in particular are difficult to establish. As a result, the design of D-regions are usually based on simplified modelling using the equilibrium conditions alone; the strain conditions are ignored.

In prestressed concrete structural members, the anchorage zone where the prestressing force from the stressed tendons is transferred to the concrete either by bond between steel and concrete (pre-tensioned construction) or by bearing-type anchorages (post-tensioned construction) may be considered as a disturbed or D-region. For example, let us consider the simplest case of a concentrically prestressed concrete beam. As shown in Fig. 8.1 (a), the prestressing force is transferred to the concrete through a stiff bearing plate whose depth is smaller than the overall depth of the beam. Obviously, the stress distribution across a section located at a distance sufficiently away from the end of the beam, such as section A-A in Fig. 8.1(a), is uniform, the magnitude of the stress being $F/A_c$. In contrast, close to the loaded end, where the load is applied in a concentrated fashion, the distribution of stresses is far from this ideal distribution, and cannot be so readily obtained.
Figure 8.1 Various representations of the stress distribution in an anchorage zone.

However, advanced methods, such as the finite element method or photoelastic analysis, are now available that give a better picture of the nature of stress distribution or the stress field in this region. In Figs. 8.1 (b) – (e), the distribution of stresses in a vertical plane through the centre of anchorage, as obtained from such analyses, are plotted qualitatively in a variety of ways. A similar situation also exists in a horizontal plane through the centre of anchorage if the width of the bearing plate is smaller than the width of the beam. It may be seen that transverse tensile and compressive stresses of considerable magnitude occur in this region, and that a finite length is necessary for this localised disturbance to subside. The length of the beam measured from the loaded end beyond which stress distribution becomes linear and readily predictable is known as the lead length. For pre-tensioned members, the lead length (or the D-region) equals the prestress transfer length, whereas in post-tensioned construction, it is approximately equal to the longer dimension of member cross section.
In this Chapter, design considerations for this disturbed region at the anchorage zone will be presented. Treated first is the anchorage zone for post-tensioned concrete beams. In pre-tensioned members, the condition is less severe because the load is introduced more gradually over the transfer length. The main features of the anchorage zone in such members will be highlighted later in the chapter.

8.2 ANCHORAGE ZONE FOR POST-TENSIONED BEAMS

In post-tensioned members, the prestressing force from the tendon is transferred to the concrete essentially by direct bearing through the anchorage. The anchorage systems used are proprietary items. The basic components of such an anchorage system are shown in Fig. 8.2. In general, these units are recessed into the end of the member. Some anchorage devices are provided with fins to assist in distributing the large concentric force. Spiral reinforcement often forms part of the anchorage system. Located immediately behind the anchorage, the spiral reinforcement enhances the bearing capacity of the concrete by providing confining action.

Being proprietary, the anchorage devices are standardised. The bearing plates of these standard anchorage systems are specially designed so that the bearing stresses they impose on the concrete are less than the permissible bearing stress. Therefore, in design it is normally unnecessary to consider the bearing failure, provided that the concrete attains the desired level of maturity at transfer of prestress and is adequately compacted in the anchorage zone.

Figure 8.2 Components of a typical anchorage system.

Similarly, compressive and shear stresses that occur within the lead length in a general situation, though not insignificant, do not cause any concern. But tensile stresses induced at right angles to the beam axis often exceed the tensile strength of the concrete. If sufficient reinforcement were not provided, visible longitudinal cracks would appear at the ends of the beam. As shown respectively in Figs. 8.3(a) and (b), these cracks may be classified as bursting and splitting cracks. It should be noted here that the purpose of the reinforcement is to contain such cracks rather than to stop them forming. In this section, the transverse tensile stresses induced within the lead length, and the design of the necessary steel reinforcement to restrict the growth of cracks will be considered.
Fig. 8.3 Typical longitudinal cracking of a beam with inadequate transverse reinforcement.

8.2.1 End Block

In post-tensioned construction, it has become customary to enlarge the beam web to a width equal to the width of the smaller flange in case of an I-section, or fill the void for a box section at the ends of the beam as shown in Fig. 8.4. Such a segment of the beam with enlarged concrete area at its each end is known as end block.

Fig. 8.4 Typical end block for a post-tensioned symmetrical I-beam.

Although, tests have shown that provision of an end block does not help in preventing the formation of cracks, they are often necessary for practical reasons, such as

- To accommodate anchorage devices
- To provide a smooth transition for the strands
- To allow for a practical distribution of anchorages or bearing plates at the ends of the beam
To furnish adequate bearing area for the beam at the supports and at anchorage points, and
To facilitate detailing of reinforcement without creating congestion.

For most practical design, the length of the end block is taken equal to the overall depth of the beam.

8.2.2 Analysis and Design Considerations

It has been mentioned that a complex three-dimensional stress field exists within an end block, and that the main design concern is the tensile stresses. The tensile stress field is such that splitting or bursting cracks tend to appear in a direction parallel to the longitudinal axis of the beam, and this is accompanied by spalling of concrete at corners of the loaded face. An accurate determination of the magnitude and distribution of these stresses is quite complex and often unnecessary for practical design. Experiences have shown that a satisfactory design can be achieved by using simple two-dimensional modelling of the actual stress field.

In the design of an end block, the tensile strength of the concrete is generally ignored. Therefore, there is no need to calculate the maximum tensile stress to check whether the beam will crack or not. It is however necessary to make a reasonable estimate of the tensile stress resultant, and the region within which these stresses will act. Knowing the tensile stress resultant, the total amount of reinforcement necessary to carry this tension without creating a crack that is unacceptable from serviceability viewpoint may be calculated. This is accomplished by setting a limit to the steel stress on the basis of empirically established relationships between steel stress and crack width. The total amount of reinforcement is then distributed within the tension zone, the extent of which has to be fixed from experience, and detailed in a suitable manner. Patience and engineering judgement are the essential ingredients for achieving a simple, but satisfactory detailing of reinforcement in an end block.

From the preceding discussion, it becomes obvious that the analysis of an end block basically means finding the magnitude of the tensile stress resultant. Two simple models are available to accomplish this task. These are beam analogy model and strut-and-tie model. The code also provides some guidelines to deal with simple cases. The prestressing force considered in the analysis is the maximum jacking force and the vertical component of prestress at the support is usually neglected. Since the analysis and design of a centrally loaded end block provide the bases for dealing with more complex cases, this will be treated first in detail.

8.3 END BLOCK WITH A CONCENTRIC ANCHORAGE

8.3.1 Analysis by Beam Analogy

A simple model for estimating the tensile stress resultant is to consider the end block as a beam supported on one side by the bearing plates of the anchorage devices and loaded on the other side by linearly distributed stresses in the beam's main region. First, the shear force and bending moment diagram for the idealised beam are drawn, and the peak moments identified. From elementary mechanics, peak
moments will occur at sections where shear force changes sign. The tensile and compressive stress resultants at each of these sections may then determined from statics provided the lever arm between them is known. Some approximations or guidelines are needed to establish the lever arm and the extent within which the tensile stresses act.

Fig. 8.5(a) shows a real end block containing a single, centrally placed anchorage with a bearing plate of height, h. The beam analogy model together with the shear force and bending moment diagrams for the end block is shown in Fig. 8.5(b). It may be seen that the maximum moment occurs along the line of action of the prestressing force (axis of the anchorage). This moment is denoted by \( M_b \). The magnitude of \( M_b \) may be easily obtained from rotational equilibrium of the free-body diagram shown in Fig 8.5 (c). Taking moments about the axis A-A, we obtain

\[
M_b = \frac{F \left(D - \frac{h}{4}\right)}{2} = \frac{F}{8} (D - h) \tag{8.1}
\]

![Figure 8.5 Beam analogy. (a) Real end block, (b) Analogous beam, and (c) Free-body diagram.](image)

The moment \( M_b \) is caused by the compressive stress resultant, \( C_b \) and the tensile stress resultant, \( T_b \). As can be seen in Fig. 8.5(a), \( T_b \) acts away from the loaded face, which tends to cause bursting of the beam, as shown. Hence, \( M_b \) is known as the bursting moment and \( T_b \) as the bursting tension. For equilibrium,

\[
T_b = C_b \tag{8.2a}
\]

and

\[
M_b = T_b \ell \tag{8.2b}
\]

where \( \ell \) is the lever arm between \( T_b \) and \( C_b \).
Analytical and experimental investigations have shown that, except for very small and rarely used concentration ratios \((h/D < 0.15)\), the lever arm \(\ell\) may be reasonably approximated by \(D/2\), that is, half the depth of the end block. Thus, substituting the value of \(M_b\) from Eq. (8.1) into Eq. (8.2b) and setting \(\ell = D/2\), we obtain \(T_b\) as

\[
T_b = \frac{2M_b}{D} = \frac{F}{4} \left(1 - \frac{h}{D}\right)
\]

(8.3)

In the above analysis, only the vertical tension that occurs in an end block has been considered. As shown in the end view of an anchorage zone in Fig. 8.6(a), tension in the horizontal direction will also exist if the width of the bearing plate is smaller than that of the end block. The magnitude of this horizontal bursting tension may be obtained by considering the beam lying on one of its sides, as shown in Figs. 8.6(b) and (c), and analysing the end block in the same way as that for vertical tension or, in other words, by replacing \(h\) by \(h'\) and \(D\) by \(b\) in Eqs. (8.1) and (8.3).

Figure 8.6 Analysis for horizontal bursting tension. (a) End view of the end block, (b) Orientation of the beam, and (c) Real end block.

### 8.3.2 Strut-and-Tie Model

An alternative model for estimating the tensile stress resultant is to assume that in a cracked member, concrete in between the cracks carries direct compression and steel carries axial tension. The load-carrying mechanism of the member can then be idealised as that of a truss comprising a series of concrete struts and steel ties. Known as the strut-and-tie model, this concept of idealisation has been found to be a simple, but powerful tool in dealing with disturbed or discontinuous regions of a concrete structure.
A simple strut-and-tie model for the D-region of the beam in Fig. 8.1 (a) is shown in Fig. 8.7. The procedure for setting up a suitable model may be briefly described as follows:

1. Isolate the D-region (in this case the anchorage zone), and consider it as an element. The region may be assumed to extend a distance $D$ from the loaded end, where $D$ is the overall depth of the beam.

2. Calculate the internal stresses on the boundaries of the element by elastic theory. In the present case, the stress distribution on the right-hand side of the element is uniform with an intensity of $F/D$.

3. Subdivide the boundary and obtain the stress resultant on each sub-length. In the model presented in Fig. 8.7, the right-hand boundary has been subdivided into two, the resultant force in each being $F/2$. Several other strategies may also be followed, which will become evident from the subsequent discussion. For example, the loaded area on the left-hand boundary may also be subdivided into two, as shown in an alternative strut-and-tie model for a centrally loaded end block in Fig. 8.8.

4. Draw a truss to transmit the forces from one boundary to the other noting that when two non-collinear forces meet at a point, a third force is necessary to maintain equilibrium of the joint. For example, the two compression members meet at joint B in Fig. 8.7. Since they have different inclinations, a tension tie BC is needed for equilibrium. Also, some judgement is necessary to fix the inclination of the members AB and AC. In most cases, a slope of about 2:1 can be assumed.

5. Find the forces in each member of the resulting truss. Once the forces are known, the truss members may be designed or checked for adequacy.

Figure 8.7 A simple strut-and-tie model.

Figure 8.8 An alternative strut-and-tie model for an end block with a central anchorage.
It has been mentioned that in the design of an end block, the designer is most concerned with the tension forces that exist in the region. By setting up the strut-and-tie model, it is thus a simple matter to locate the position of the tension tie, and the magnitude of the tension force. The design can then follow the usual routine.

### 8.3.3 Code Approach

British Code provides only a brief guideline for the analysis and design of rectangular end blocks for serviceability. It states that the bursting tensile force, $T_b$, in an individual end block loaded by a symmetrically placed square bearing plate depends on the concentration ratio, $h/D$, and its magnitude may be obtained from Eq. (8.4).

\[
\begin{align*}
\text{when } 0.2 \leq h/D \leq 0.3, & \quad T_b = 0.23F_j \\
\text{when } 0.3 < h/D \leq 0.7, & \quad T_b = \left(0.32 - 0.3 \frac{h}{D}\right)F_j
\end{align*}
\]  

(8.4a) \hspace{1cm} (8.4b)

in which $F_j$ is the jacking force. Both vertical and horizontal bursting tension should be assessed in relation to $h/D$ for the respective direction. When circular bearing plates are used, they should be replaced by square plates of equivalent area. When several anchorages are involved, the end block should be divided into a series of symmetrically-loaded prisms and each prism treated in the above manner. Without further elaboration, the Code directs the designer to specialist literature.

### 8.3.4 Design of Reinforcement

#### Bursting tension

Once the tensile stress resultant is found, the amount of reinforcement is calculated by assigning a maximum stress level, $f_s$, in the steel reinforcement to restrict the growth of cracks. Thus, the total amount of reinforcement required is

\[
A_s = \frac{T_b}{f_s}
\]

(8.5)

Australian Code, AS 3600-1988, which utilises the beam analogy model to develop design provisions, suggests that steel stress of no more than 150 MPa should be used for crack control. This reinforcement should be furnished by closed links or spirals and distributed uniformly in a region extending from $0.2D$ to $D$ from the loaded face. Therefore, the area of reinforcement per unit length of the end block required for bursting tension is

\[
\frac{A_{sv}}{s_v} = \frac{A_s}{0.8D}
\]

(8.6)

where $s_v$ is the spacing of closed links and $A_{sv}$ is cross-sectional area of links per spacing, $s_v$. 
Eq. (8.6) may be reorganised to obtain the spacing of links, for an assumed size (bar diameter) and number of legs, as

\[ s_e = \frac{0.8DA_e}{A_g} \]  

(8.7)

The stirrup size and spacing so determined should also be provided in the portion of the end block from 0.2D to as near as practicable to the loaded face satisfying the clear concrete cover requirement.

BS8110-1997, on the other hand, recommends that the steel stress for crack control be taken as 200 MPa and that the reinforcement be distributed uniformly in a region extending from 0.1D to D from the loaded face.

For bursting tension, it is a good practice to use two sets of links, one enclosing the anchorage itself and the other to enclose the entire section, as shown in Fig. 8.9. Additional longitudinal bars must be placed at each corner anchorage of links if they are not available from the design of the main region. While detailing, care must be taken to ensure that all potential cracking planes are reinforced, and that sufficient space is available for the concrete to flow through the reinforcement assembly for adequate compaction.

![Diagram](image)

**Figure 8.9** Detailing of reinforcement for a centrally loaded end block.

**Surface spalling tension**

It can be seen in Fig. 8.1 that in addition to bursting tension well inside the end block, tensile stresses of considerable magnitude also occur close to the surface. Known as surface spalling tension, these stresses may lead to a beam without any corners unless properly reinforced to prevent spalling of the concrete. For a centrally loaded end block with a concentration ratio \((h/D)\) of 0.1 or more, the surface spalling tension, \(T_{sa}\) may be taken as 0.03\(F\). In the case of multiple anchorage, the surface tensile force between two anchorages may be estimated as \(0.5F\), where \(F\) is the sum of the prestressing force at the two adjacent anchorages.

Once \(T_{sa}\) is estimated, the required area of reinforcement may be calculated from Eq. (8.5) and is distributed parallel to the loaded face with adequate concrete protection, as shown in Fig. 8.9. Welded wire fabric may be a suitable option for furnishing reinforcement for surface spalling tension.
EXAMPLE 8.1
A horizontal cable comprising 14 strands of 15.7 mm diameter is anchored at the centre of a rectangular end block 400 mm wide and 1000 mm deep using a 260 mm square bearing plate, as shown in Fig E8.1.1. The maximum jacking force at transfer of prestress is 2800 kN. The following information is given:

\[ f_a = 35 \text{ MPa}, \quad f_{cu} = 50 \text{ MPa}, \quad f_s = 150 \text{ MPa} \]

Design the end block reinforcement using the beam analogy model.

SOLUTION

(a) Design for vertical bursting tension

This is a standard case. Therefore, Eqs. (8.3) can be directly applied to obtain the magnitude of vertical bursting tension behind the anchorage as

\[ (T_b)_v = \frac{2M_b}{D} \cdot \frac{E}{A} \left(1 - \frac{h}{D}\right) = \frac{2800}{4} \times \left(1 - \frac{260}{1000}\right) = 518 \text{ kN} \]

The total amount of reinforcement required for vertical bursting tension is

\[ A_s = \frac{A_{(T_b)_v}}{f_s} = \frac{518,000}{150} = 3453 \text{ mm}^2 \]

over a length of 0.8D = 800 mm. Assuming 4 legs of T12 links, the required spacing is

\[ s_v = \frac{0.8DA_{sv}}{A_s} = \frac{0.8 \times 1000 \times 4 \times 113}{3453} = 104 \text{ mm} \]

Use 4 vertical legs of T12 @ 100 mm up to a distance of 1000 mm from the loaded face.

(b) Design for horizontal bursting tension

in this case, the concentration ratio $h/D' = H'/b = 250 / 400 = 0.625$. Therefore, Eq (8.3) gives

\[ (T_b)_h = \frac{2M_b}{D} \cdot \frac{E}{A} \left(1 - \frac{h}{D}\right) = \frac{2800}{4} \times \left(1 - 0.65\right) = 245 \text{ kN} \]
which requires an area of transverse reinforcement, \( A_s = 245,000 / 150 = 1633 \text{ mm}^2 \). This is to be distributed over a length of \( 0.8b = 320 \text{ mm} \). Assuming four horizontal legs, 2 of T12 (full depth links) and 2 of T16 (enclosing the anchorage) links, the required spacing is

\[
 s_v = \frac{0.8bA_s}{A_s} = \frac{0.8 \times 400 \times (2 \times 113 + 2 \times 201)}{1633} = 123 \text{ mm}
\]

Keeping in view the reinforcement selected for vertical bursting tension, we shall choose a spacing of 100 mm for the sake of construction simplicity.

(c) Surface spalling tension

The surface spalling tensile force for a centrally loaded end block may estimated as \( 0.03F = 0.03 \times 2800 = 84 \text{ kN} \). This requires a reinforcement area \( A_s = 84,000 / 150 = 560 \text{ mm}^2 \). Noting that the first stirrup placed for carrying bursting tension will also be available for surface palling tension, we select a welded wire fabric of 8 mm wire diameter and 100 mm square grid. This fabric should be placed outside the first stirrup.

(d) Design summary

<table>
<thead>
<tr>
<th>Design consideration</th>
<th>Reinforcement requirement</th>
<th>Spacing</th>
<th>Distance from the loaded face</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical bursting tension</td>
<td>4 V legs of R12</td>
<td>100 mm</td>
<td>1000 mm</td>
</tr>
<tr>
<td>Horizontal bursting tension</td>
<td>2 H legs of R12</td>
<td>100 mm</td>
<td>400 mm</td>
</tr>
<tr>
<td></td>
<td>2 H legs of R16</td>
<td>100 mm</td>
<td>400 mm</td>
</tr>
<tr>
<td>Surface spalling tension</td>
<td>T8</td>
<td>100 mm sq. grid</td>
<td>Next to loaded face</td>
</tr>
</tbody>
</table>

(e) Reinforcement detailing

The reinforcement details shown in Fig. E8.1.2 have been arrived at after a careful review of the reinforcement requirements for bursting and surface spalling tension, keeping in view the simplicity of construction.

(a) Cross-section

(b) Longitudinal section

**Fig. E8.1.2** Reinforcement details.
8.4 END BLOCKS WITH AN ECCENTRIC ANCHORAGE

8.4.1 Beam analogy

In the case of a centrally loaded end block, that is, when \( e_0 \) is zero, the stress distribution at the end of the lead length is obviously uniform. When the load is applied eccentrically, its effect at the end of the lead length may be replaced by a force-couple system acting at the centroid of the section as discussed in Chapter 2. The axial force, \( F \), will produce uniform compression, while the couple, \( F e_0 \), will generate linearly varying stresses, tension on one side and compression on the other. For small eccentricity, the resulting stress distribution obtained by linear superposition of these stresses will be trapezoidal. As the eccentricity is increased, a stage will reach when the trapezoidal distribution will reduce to a triangle, as shown in Fig. 8.10. The corresponding eccentricity of the applied force is known as the kern distance and, for a rectangular section, this easily works out to be \( D/6 \).

The beam analogy model for this particular case, as constructed in Fig. 8.10, indicates that the shear force changes sign only once. This means that still there is only one peak in the bending moment diagram and this occurs approximately along the line of action of the prestressing force.

![Beam analogy model for an end block with \( e_0 = D/6 \)](image)

Figure 8.10 Beam analogy model for an end block with \( e_0 = D/6 \)

As the eccentricity is increased beyond \( D/6 \), the top fibres will be subjected to tension. The beam analogy model for such a case is presented in Fig. 8.11 which clearly shows that the shear force changes sign twice. This means that there are two peaks in the bending moment diagram. The first peak occurs close to the line of application of the prestressing force. Denoted by \( M_b \), this moment will generate bursting tensile stresses well inside the anchorage point, as shown. The other peak moment, as denoted here by \( M_e \), acts in a sense opposite to \( M_b \). As shown in the free-body diagram of Fig. 8.11(b), this moment will cause tension close to the loaded face, which will tend to split the beam as shown in Fig. 8.11(a). Knowing the prestressing force and its eccentricity, the magnitudes and locations of \( M_e \) and \( M_b \) can be easily determined from statics. It should be noted here that the maximum \( M_b \) does not necessarily occur along the line of action of the prestressing force.
Figure 8.11 Beam analogy model for an end block with eccentric anchorage placed at a large eccentricity.

In order to calculate the magnitude of the tensile stress resultants, $T_b$ and $T_s$ corresponding to $M_b$ and $M_s$, respectively, it is however necessary to make a reasonable estimate of the respective lever arms between the tensile and compressive stress resultants forming the resisting couples.

**Lever arm for bursting tension**

In case of bursting tension, the isobars presented in Fig. 8.12 for a rectangular end block with an eccentric anchorage shows that the lead length is greatly reduced, and this reduces the lever arm from that obtained for a centrally loaded end block.

Fig. 8.12 Isobars for an eccentrically loaded rectangular end block.
A simple method to estimate the reduced lever arm for an eccentrically loaded end block is provided by the symmetrical prism concept proposed by Guyon. According to the method, the stresses in a real end block are approximately the same as those in an imaginary end block comprising a prism whose centre line is the line of action of the prestressing force, \( F \), and whose depth is twice the distance from the position of \( F \) to the nearest free edge of the beam. Fig. 8.13 shows the symmetrical prisms of depth, \( D_s \) for two cases of eccentrically loaded end blocks.

![Symmetrical prism for eccentrically loaded rectangular end blocks.](image)

For the purpose of analysis and design of the real end block, the symmetrical prism is treated exactly the same way as it was done for a centrally loaded end block. The lever arm \( C_s \) and \( T_b \) is then obtained as \( D_s / 2 \) yielding

\[
T_b = \frac{2M_b}{D_s} \tag{8.8}
\]

in which \( M_b \) is the maximum bursting moment behind the anchorage for the real end block [see Fig. 8.11 (c)].

**Lever arm for splitting tension**

For splitting moments, the lever arm between the resultant transverse tension \( T_s \) and compression \( C_s \) is usually larger than for bursting tension, as can be seen in the isobars of Fig. 8.12. According to the Australian code, AS 3600-1988, the transverse tension at the loaded face remote from the anchorage for a single eccentric anchorage may be calculated by assuming that the lever arm is half the overall depth of the member. Thus, for a single eccentric anchorage

\[
T_s = \frac{2M_s}{D} \tag{8.9}
\]

in which \( M_s \) may be obtained from the free-body diagram of Fig. 8.11(b).
8.4.2 Strut-and-Tie Model

The strut-and-tie model for an end block with a single eccentric anchorage may be constructed as shown in Fig. 8.14.

![Diagram](image)

Fig. 8.14 Strut-and-tie model for eccentrically loaded end block.

**Example 8.2**

A horizontal cable comprising 7 strands of 15.7 mm diameter is anchored in a rectangular end block 400 mm wide and 1000 mm deep using a 220 mm square bearing plate placed at an eccentricity of 300 mm below c.g.s., as shown in Fig. E8.2.1. The maximum jacking force at transfer of prestress is 1400 kN. The following information is given:

\[ f'_{c} = 35 \text{ MPa}; \quad f_{cu} = 50 \text{ MPa}; \quad \sigma_s = 150 \text{ MPa} \]

Design the end block reinforcement.
SOLUTION

The location and dimensions of the symmetrical prism, together with the analogous beam, are shown in Fig. E8.2.2.

(a) Stress distribution at the end of the lead length

**Top fibre**

\[ \sigma_{\text{top}} = \frac{F}{A_c} - \frac{F_{e_y}y_1}{l} = \frac{1400 \times 10^3}{400 \times 1000} - \frac{1400 \times 10^3 \times 300 \times 500 \times 12}{400 \times 1000^3} \]

\[ = 3.50 - 56.30 = -2.80 \text{ MPa} \]

\[ = 2.80 \times 400 = 1.12 \text{ kN/mm} \]

**Bottom fibre**

\[ \sigma_{\text{bot}} = \frac{F}{A_c} + \frac{F_{e_y}y_1}{l} = 3.50 + 6.30 = 9.80 \text{ MPa} \]

\[ = 9.80 \times 400 = 3.92 \text{ kN/mm} \]
(b) Location of $M_c$ and maximum $M_b$

Distance $x_1$ (Refer to Fig. 8.2.2)
Since the top fibres are subjected to tension, splitting moment will exist near the loaded face, in addition to bursting moment behind the anchorage. To obtain the magnitude of the splitting moment, $M_s$, we have to find the distance from the top fibre where shear force in the analogous beam changes sign. In this case, this depth equals to twice the distance from the top fibre where longitudinal stress is zero. The distance, $y$, from c.g.c. where stress is zero is obtained as follows:

$$\frac{F}{A_c} = \frac{F_{c,y}}{l} = 3.50 \times \frac{1400 \times 10^3 \times 300 \times y \times 12}{400 \times 1000^3} = 0$$

which gives $y = 278$ mm

Thus, the distance, $x_1$, from the top fibre (Refer to Fig. 8.1) where shear force in the analogous beam changes sign is

$$x_1 = 2 \times (500 - 278) = 444 \text{ mm}$$

Distance $x_2$ (Refer to Fig. 8.2.2)
The maximum bursting moment, $M_b$, occurs at a distance $x_2$ from the bottom face where shear force in the analogous beam is zero. Referring to Fig. 8.2.2, the shear force at a distance $x_2$ may be obtained as

$$V = 6.36 \left(x_2 - 90\right) \left[\frac{3.92 + (3.92 - 0.050x_2)}{2}\right] x_2$$

Equating this to zero, we obtain $x_2 = 195.4$ mm

(b) Design for bursting tension

Vertical bursting tension
Consider the free-body diagram of the segment of the end block below the centre line of the anchorage, as shown in Fig. E10.2.2.

![Figure E8.2.2.](image-url)
Taking moment of horizontal forces about the line A-A, we obtain

\[ M_b = (2.935 \times 195.4 \times 97.7 + 0.5 \times 0.985 \times 195.4 \times 130.3) \times 10^{-3} - 670 \times 52.7 \times 10^{-3} = 33.3 \text{ kN.m} \]

Therefore

\[ T_b = \frac{2M_b}{D_s} = \frac{2 \times 33.3}{0.4} = 166.5 \text{ kN}; \quad A_s = \frac{T_b}{\sigma_s} = \frac{166.5 \times 10^3}{150} = 1110 \text{ mm}^2 \]

Assuming 4 legs of T10 links, the required spacing is

\[ s_v = \frac{0.8D_s A_{cv}}{A_s} = \frac{0.8 \times 400 \times 4 \times 78.5}{1110} = 90 \text{ mm} \]

Use 4 vertical legs of T10 links @ 90 mm up to a distance of 400 mm from the loaded face.

**Horizontal bursting tension**

For horizontal bursting tension, replacing \( D \) by \( b = 400 \text{ mm} \) and \( h \) by \( h' = 220 \text{ mm} \) in Eq. (8.3), we obtain

\[ T_b = \frac{1400}{4} \left( 1 - \frac{220}{400} \right) = 157.5 \text{ kN}; \quad A_s = \frac{T_b}{\sigma_s} = \frac{157.5 \times 10^3}{150} = 1050 \text{ mm}^2 \]

Assuming 4 legs of T10 links, the required spacing is

\[ s_v = \frac{0.8 \times 400 \times 4 \times 78.5}{1050} = 95 \text{ mm} \]

Use 4 horizontal legs of T10 links @ 90 mm up to a distance of 400 mm from the loaded face. **Note that the above calculation is based on the average stress.**

(b) Design for splitting tension

The free-body diagram of the segment of the end block above the line where shear changes sign is shown in Fig. E8.2.3.

![Diagram](image)

Figure E8.2.3.
Taking moment about B-B, we obtain
\[ M_y = 124.3 \times \frac{2}{3} \times 0.444 = 36.8 \text{ kNm} \]
Thus,
\[ T_s = \frac{2M_y}{D} = \frac{2 \times 36.8}{1} = 73.6 \text{ kN}; \quad A_y = \frac{T_s}{\sigma_y} = \frac{73.6 \times 10^3}{150} = 491 \text{mm}^2 \]

Use 2 vertical legs of T16 and 2 vertical legs of T10 links close to the loaded face, \( A_y \) provided = 559 mm².

(c) Design for surface spalling tension
The surface spalling tension is estimated as \( 0.03F = 0.03 \times 1400 = 42 \text{ kN} \). This requires a reinforcement area of
\[ A_y = \frac{42 \times 1000}{150} = 280 \text{mm}^2 \]
The links provided for splitting tension already supplied an area of 559 mm². Hence, no additional reinforcement is necessary. However, because of the eccentric nature of the anchorage, a layer of weided wire fabric with 7 mm wire diameter and 100 mm square grid placed next to the first link may be considered.

(d) Reinforcement details
The final arrangement of reinforcement satisfying the requirements of bursting, splitting and surface spalling tension simultaneously is shown in Fig. E8.2.4.

![Diagram](image)

**Figure E8.2.4**

### 8.5 END BLOCK WITH TWO OR MORE ANCHORAGES

When an end block contains two or more anchorages, the analysis may be performed using the beam analogy model as illustrated in the preceding article or by
constructing a strut-and-tie model using the principles laid down earlier for a single anchorage. In this section, only the beam analogy model will be discussed.

Fig. 8.15 shows the beam analogy model for an end block that contains two symmetrically placed bearing plates. It may be seen that there are three peaks in the bending moment diagram for the analogous beam. Two of these peaks generate bursting tension behind each anchorage, as shown. The remaining peak moment occurs in between the two anchorages and, depending on the distance between them, it produces either splitting (when they are far apart) or bursting tension (when the anchorages are placed close to each other). The magnitudes of these moments may be obtained from the free body diagrams of appropriate segments of the end block as shown in Fig. 8.15. To obtain the associated bursting (or splitting) tension, it is however necessary to estimate the lever arm between the compressive and tensile stress resultants for each case. The method of successive resultants developed by Guyon together with the concept of symmetrical prism provides a simple means to accomplish this.

Fig. 8.15 Beam analogy model for end blocks with two symmetric anchorages.

When multiple anchorages are involved, the depth of symmetrical prism associated with a particular anchorage, when all anchorages are stressed, is taken as the smaller of the
(a) distance from the centre of the anchorage to the centre of the nearest anchorage, and
(b) twice the distance from the centre of the anchorage to the nearest free edge of the end blocks.

According to the method of successive resultants, transverse stress distributions must be considered in three different steps. First, along the line of action of each individual force in a group, then on the line of action of the resultant of each group and, finally, along the line of action of the total resultant. At each step, symmetrical prisms are considered in accordance with the above guidelines. The symmetrical prism appropriate to an anchorage or a group of anchorages is then used to determine the tensile force and the extent of the zone of tension in a manner similar to an end block with a single anchorage. That is, the lever arm between $C$ and $T$ is taken as half the depth of respective symmetrical prism. Thus, the treatment of an end block with multiple anchorages reduces to that of a series of end blocks, each containing a single anchorage.

To illustrate the method, let us consider the end block of Fig. 8.16. In this case, the end block needs to be analysed in two steps, first along the line of action of each force, and then along the line of action of their resultant. The symmetrical prisms appropriate for the two successive steps are shown respectively in Figs. 8.16(a) and 9(b).

![Figure 8.16 Method of successive resultant for an end block with two symmetrical anchorages.](image)

As a further illustration to the method of successive resultants, the steps involved in the analysis of an end block with three anchorages, in which two of them form a group, are shown in Fig. 8.17. This scheme is applicable only when all the cables have been stressed and anchored.
Figure 8.17 Method of successive resultant for an end block with multiple anchorages.

In the case of an end block with multiple anchorages, it should be noted that stressing of the cables is usually done one after the other in sequence. At the end of each stressing operation, the end block should be investigated for the transverse tensile stresses and sufficient reinforcement provided to take care of them. Since, some intermediate stage may become more critical than the final stage when all the cables have been stressed, the designer should carefully consider the stressing sequence and integrate them in design. He should then specify the sequence of tensioning operation conforming to what he had considered in obtaining that particular solution. For the end block in Fig. 8.17, it is obvious that a total of three different stages of prestressing need to be considered. One suitable scheme for the sequence of tensioning is shown in Fig. 8.18.

Figure 8.18 Sequence of tensioning the cables.
8.6 ANCHORAGE ZONE FOR PRE-TENSIONED BEAMS

8.6.1 TRANSFER (TRANSMISSION) LENGTH

In the case of pre-tensioned beams, the prestress is transferred to the concrete by bond at the ends of the beam. Therefore, at each end of the beam there exists a length, known as the transfer length or transmission length, $L_t$, where bond stress is high and in which prestress is transferred gradually from zero to its full value. The nature of bond in the transfer length of pre-tensioned beams differs from normal bond by adhesion, friction and mechanical action (in case of deformed bars or wires). When a tendon is released, the bond stress at the very end of the beam is extremely high and local bond failure occurs. Consequently, the tendon slips and sinks into the concrete and loses stress so that it expands. In doing so, the tendon wedges itself against the concrete in a manner shown in Fig. 8.20, so that the bond within the transfer length is due mostly to this wedging action.

![Diagram of transfer length and bond stress](image)

Figure 8.18 Transfer of prestress by bond in pre-tensioned beams.

The actual transfer (transmission) length in a pre-tensioned beam depends on a number of factors, such as

(a) degree of compaction of the concrete
(b) size and type of tendons
(c) strength of the concrete
(d) deformation and surface condition of the tendon.

BS 8110-1997 specifies that $L_t$ should be based on experiments as far as possible. However, in the absence of experimental evidence, the following equation may be used to estimate $L_t$:

$$L_t = \frac{K_t d_b}{\sqrt{f_{ct}}}(8.10)$$

where

- $f_{ct}$ = concrete strength at transfer
- $d_b$ = nominal diameter of the tendon
- $K_t$ = a coefficient dependant on the type of tendon
  - 600 for plan or indented wires
  - 400 for crimped wire
  - 240 for 7-wire stranded or super strand
  - 360 for 7-wire drawn strand

8.6.2 DESIGN CONSIDERATIONS

The design of pre-tensioned members as affected by the transfer length may be described in three parts as follows:

(a) The presence of a zone of low prestress at the ends of a beam must be considered in design, particularly in the design for shear.
(b) Premature failure due to bond slip can occur if flexural cracks appear near the transmission length. Therefore, no cracking should be allowed within a distance equal to $2L_t$ from the end of the tendons before final collapse.
(c) Although the prestress is not concentrated at the free end due to gradual transfer along a certain length, high stress concentration may occur little inside the free end. Therefore, end zones of pre-tensioned members must be additionally reinforced with vertical stirrups to limit the opening of cracks. However, BS code does not give any guidance how to determine such reinforcement. Based on the study conducted at the Portland Cement Association Laboratory by Marshall and Mattock, the following equation may be suggested:

$$A_{sv} = 0.021 \frac{F_i D}{\sigma_s L_t}(8.11)$$

where $A_{sv}$ is the area of vertical stirrups and $\sigma_s$ is the permissible stress in steel which may be taken as 150 MPa. The transverse steel $A_{sv}$ should be uniformly distributed over a distance $0.2D$ from the end of the beam.
EXAMPLE 8.3
Two horizontal cables, each comprising 7 strands of 15.7 mm diameter, are symmetrically anchored in a rectangular end block 400 mm wide and 1000 mm deep. The anchorages have 220 mm square bearing plates and are placed 600 mm apart as shown in Fig. E8.3.1. The maximum jacking force at transfer of prestress is 1400 kN in each cable. The following information is given:

\[ f_{ci} = 35 \text{ MPa}; \quad f_{cu} = 50 \text{ MPa}; \quad f_s = 150 \text{ MPa} \]

Assuming that the dimensions of the bearing plates are adequate to exclude bearing failure of the concrete, design the reinforcement for the end block.

Figure 8.3.1

SOLUTION

In designing this end block, two stages of prestressing need to be considered. Stage 1 is during stressing of the first cable, and Stage 2 is when both the cables have been stressed.

Stage 1: When one cable is stressed

Let us specify that the bottom cable be stressed first. The problem then becomes identical to the one that has been dealt with in Example 8.2. The reinforcement requirement that has been arrived at in Example 8.2 should also apply to this end block at this stage of prestressing.

Stage 2: When both the cables are stressed

(a) Vertical Direction
Step 1: Vertical bursting tension behind each anchorage
The dimensions of the symmetrical prisms and the distribution of the normal stresses at the end of the lead length are shown in Fig. E8.3.2(a). Obviously, the maximum bursting moment occurs at the level of zero shear in the analogous beam. Because of symmetry, it occurs at the same distance, \( x \) mm, from the top and
bottom faces of the end block, as shown in Fig. E8 3.2(a). This distance may readily be obtained from

$$6.36 \left( x - 90 \right) - 2.8x = 0$$

which gives $$x = 160.8 \text{ mm}$$

Taking moment about B-B of the free-body diagram of the end block shown in Fig. E8.3.2(b), we obtain

$$M_b = 450.3 \times (80.4 - 35.4) = 20,300 \text{ kNmm}$$

Therefore,

$$T_b = \frac{2M_b}{D_s} = \frac{2 \times 20,300}{400} = 101.5 \text{ kN}$$

This is smaller than the value for $$T_b$$ when only the bottom cable was stressed. Since the same symmetrical prism is also applicable here, the reinforcement provided in stage 1 will be adequate to take care of the tension behind the anchorage in stage 2.

Step 2: Vertical tension along the line of action of the resultant, that is, at the centre of end block.

The free-body diagram of the end block through the line of action of the total resultant is shown in Fig. 8.3.3. Obviously, the moment acting along this line is of splitting type.
Taking moment about C-C, we obtain

\[ M_s = 1400 \times (300 - 250) = 70,000 \text{ kNm} \]

With the depth of symmetrical prism, \( D_s = 1000 \text{ mm} \), we obtain the splitting tension as

\[ T_s = \frac{2M_s}{D_s} = \frac{2 \times 70,000}{1000} = 140 \text{ kN} \]

The area of transverse steel required close to the loaded face is then found to be

\[ A_s = \frac{T_s}{\sigma_s} = \frac{140 \times 10^3}{150} = 934 \text{ mm}^2 \]

Use 4 legs of T16 and 2 legs of T10. \( A_s \) provided is 961 mm².

(b) Horizontal direction

When the beam placed on one of its side faces, the end block may be viewed as a centrally loaded end block, as shown in Fig. 8.3.4.

\[ FIB = \frac{2800}{400} = 7 \text{ kN/mm} \]

The bursting tension behind the anchorage may then be obtained by Eq. (8.3). Using \( h = 220 \text{ mm} \) and \( D = 400 \text{ mm} \), we obtain
$$T_p = \frac{F}{4} \left( 1 - \frac{h}{D} \right) = \frac{2800}{4} \left( 1 - \frac{220}{400} \right) = 315 \text{ kN}$$

and

$$A_z = \frac{T_p}{\sigma_s} = \frac{315 \times 10^3}{150} = 2100 \text{ mm}^2$$

This area of reinforcement is to be distributed over a length of 0.8 \( b = 320 \text{ mm} \). Assuming four horizontal legs, 2 of T12 (full depth links) and 2 of T16, the required spacing is

$$s_v = \frac{0.8bA_{yw}}{A_z} = \frac{0.8 \times 400 \times (2 \times 113 + 2 \times 201)}{2100} = 95 \text{ mm}$$

Use 2 legs of T12 and 2 legs of T16 @ 90 mm up to a distance of 400 mm from the loaded end.

(c) Design for surface spalling tension

The surface spalling tension is estimated as 0.03\( F = 0.03 \times 2800 = 84 \text{ kN} \). This requires a reinforcement area of

$$A_y = \frac{84,000}{150} = 560 \text{ mm}^2$$

The links provided for splitting tension already supplied an area of 559 mm². Hence, no additional reinforcement is necessary. However, a layer of welded wire fabric with 7 mm wire diameter and 100 mm square grid placed next to the first link may be considered.

(d) Design summary

<table>
<thead>
<tr>
<th>Stage</th>
<th>Design consideration</th>
<th>Reinforcement requirement</th>
<th>Spacing</th>
<th>Distance from the loaded face</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom cable stressed only</td>
<td>V. bursting tension</td>
<td>4 V legs of T10</td>
<td>90 mm</td>
<td>400 mm</td>
</tr>
<tr>
<td></td>
<td>V. splitting tension</td>
<td>2 V legs of T16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 V legs of T10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>H. bursting tension</td>
<td>4 H legs of T10</td>
<td>90 mm</td>
<td>400 mm</td>
</tr>
<tr>
<td>Both cables are stressed</td>
<td>V. bursting tension</td>
<td>4 V legs of T10</td>
<td>90 mm</td>
<td>400 mm</td>
</tr>
<tr>
<td></td>
<td>V. splitting tension</td>
<td>4 V legs of T16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 V legs of T10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>H. bursting tension</td>
<td>2 H legs of T16</td>
<td>90 mm</td>
<td>400 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 H legs of T12</td>
<td>90 mm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Surface spalling tension</td>
<td>T7</td>
<td>100 mm sq. grid</td>
<td>Next to loaded face</td>
</tr>
</tbody>
</table>
PROBLEMS

P8.1 A composite floor system is to be constructed using precast post-tensioned rectangular beams over a simply supported span of 15 m and a 110 mm thick cast-in-place reinforced concrete slab as shown in Fig. P8.1. It carries a characteristic imposed dead load of 2.5 kN/m² and a characteristic live load of 8 kN/m², in addition to its own weight. Flexural design at serviceability limit state yields $F_s = 1875$ kN applied at a distance of 100 mm from the soffit for the critical midspan section. This was furnished by tensioning 10 – 15.7 mm diameter strands, each with a cross sectional area of 150 mm². The prestressing tendons are contained in a single parabolic cable with eccentricities of zero at each end and 250 mm at midspan, and the anchorages have 300 mm square bearing plates. The following information is given:

For precast beam:
- $f_{cu} = 50$ MPa;
- $E_c = 30$ GPa;
- $f_{pu} = 1770$ MPa;
- $E_{ps} = 200$ GPa

For cast-in-place slab:
- $f_{cu} = 30$ MPa;
- $E_c = 26$ GPa

If the jacking force is 2000 kN, design the end block.